

國立成功大學

112學年度碩士班招生考試試題

編 號： 117

系 所： 工程科學系

科 目： 工程數學

日 期： 0207

節 次： 第 3 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Using the convolution theorem solves the equation (15%)

$$\int_0^t y(x) dx = e^{2t} \sinh t + e^t \int_0^t e^{-x} y(x) dx$$

2. Solve the initial problem of non-homogeneous ODE (20%)

$$y'' - y' - 2y = 3e^{-x} + 6e^{2x} + 2; y(0) = 1, y'(0) = 2$$

3. Using Laplace transform solves the wave equation (20%)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + a; u(0, t) = u(x, 0) = u_t(x, 0) = 0 \quad x \in [0, \infty), a \in \mathbb{R}$$

4. Suppose that there is $\mathbf{S}(u, v) = (u^2 + 1)\mathbf{i} + (v^2 - 1)\mathbf{j} + (2v^2 - u^2)\mathbf{k}$, where $0 \leq u \leq 2$ and $0 \leq v \leq 1$. Meanwhile, the velocity of water is $\mathbf{F}(x, y, z) = (x - 1)\mathbf{i} + (x + y)\mathbf{j} + (y + 1)\mathbf{k}$. Determine the amount of water flows through this surface \mathbf{S} . (15%)

5. Use the concept of the Fourier series of the function as follows: (15%)

$$f(x) = x^2 \quad (-\pi < x < \pi); f(x) = f(x + 2\pi)$$

$$\text{To find the answer of } \sum_{n=1}^{\infty} \frac{1}{n^2} = ?$$

6. Find $\int_C (z^3 - 5z + 3) dz$ (15%)

$$\text{where } C: \left\{ t + i \left(\frac{t}{i} - \sin \frac{2\pi t}{l} \right), t = 0 \rightarrow l \right\}, l < 0$$