國立高雄第一科技大學 100 學年度 碩士班 招生考試 試題紙

系 所 別：電子工程系　　　　組　別：不分組

考科代碼：1231　　　　考　科：線性代數

注意事項：

1、本科目得使用本校提供之電子計算器。

2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. Let $A=\begin{vmatrix} 2 & 3 & 5 & 9 \\ -4 & -3 & 4 & -6 \\ -4 & 0 & -3 & -8 \\ -8 & 6 & 12 & -3 \end{vmatrix}$.

   1.1 Find the rank of A. (5 points)

   1.2 Find the value of the determinant A. (5 points)

2. Let $A=\begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix}$.

   2.1 Find the inverse of A. (5 points)

   2.2 Solve the equation AX=B. (5 points)

3. Let the subspace V be generated by

   $v_1=(1,1,0,1)$, $v_2=(2,0,0,1)$, $v_3=(1,3,0,2)$,

   and the subspace W be generated by

   $w_1=(1,2,3,0)$, $w_2=(0,1,2,3)$, $w_3=(0,0,1,2)$.

   3.1 Find the dimension of $V\cap W$. (5 points)

   3.2 Find the dimension of $V+W$. (5 points)

4. Let $A=\begin{bmatrix} 2 & 1 & 1 \\ 3 & 1 & 2 \\ 7 & 3 & 3 \end{bmatrix}$. Find the LU decomposition of the matrix A. (10 points)

5. $T: R^2 \to R^3$ is a linear mapping defined by $T(x_1, x_2) = (x_1 + 2x_2, 7x_1 + 3x_2, 2x_1 + x_2)$.

5.1 Show that T is one to one. (5 points)

5.2 Show that T maps R^2 onto R^3. (5 points)

6. Let $T: M_{22} \to M_{22}$ be the linear operator defined by $T(A) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} A + A \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$.

6.1 Find the rank of T. (5 points)

6.2 Find a basis for the range of T. (5 points)

6.3 Find a basis for the kernel of T. (5 points)

7. Let $A = \begin{bmatrix} 27 & -10 \\ 34 & -5 \end{bmatrix}$.

7.1 Find the eigenvalues of A. (5 points)

7.2 For each eigenvalue of A, find an eigenvector of A corresponding to this eigenvalue. (10 points)

8. Let $T: R^3 \to R^3$ be a linear operator represented by the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 3 & 1 & -1 \\ 2 & 2 & -1 \end{bmatrix}$ with respect to the standard basis of R^3.

8.1 Find the characteristic values (eigenvalues) of T. (5 points)

8.2 Show that T is diagonalizable or not. (5 points)

9. Let $A = \begin{bmatrix} -2 & -1 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$. Find the orthogonal matrix P, such that $P^{-1}AP$ is diagonal. (10 points)