

國立成功大學

112學年度碩士班招生考試試題

編 號：112

系 所：工程科學系

科 目：線性代數與機率

日 期：0207

節 次：第 3 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) For two random variables  $X$  and  $Y$  with  $E(X) = 3$  and  $E(Y) = 10$ , please find  $E[E(X|Y)]$ .

2. (10%) Let  $A$  be an  $m \times 8$  matrix. If  $AA^T$  is an identity matrix, please provide the range of  $m$ .

3. (10%) For a  $5 \times 7$  matrix

$$H = \begin{pmatrix} 0 & 1 & & & & & \\ 0 & 2 & & & & & \\ 1 & 3 & I_5 & & & & \\ 2 & 4 & & & & & \\ 2 & 4 & & & & & \end{pmatrix}$$

where  $I_5$  is a  $5 \times 5$  identity matrix, please find a  $2 \times 7$  matrix  $G$  such that  $GH^T = \mathbf{0}$  and  $G \neq \mathbf{0}$ .

4. (20%) Let  $X$  be an exponential random variable with mean  $1/\lambda = 1$ . Let  $Y = [X] + 1$  where  $[X]$  denotes the integer part of  $X$ . (E.g.,  $[2.3] = 2$  and  $[3.9] = 3$ .) Then, please show that  $Y$  is a geometric random variable with parameter  $p$ . Also find  $p$ .

5. (10%) We roll a fair die successively. Find the expected number of rolls until the first 1 given that the first 5 occurred on the third roll.

6. (20%) Let  $X_1, X_2, \dots$  be independent and identically distributed Gaussian random variables with common mean  $\mu$  and common variance  $\sigma^2$ . If  $Y = X_1 + X_2 + \dots + X_N$  where  $N$  is a Poisson random variable with parameter  $\lambda$ , then find the moment generating function of  $Y$ .

7. (10%) Let  $X$  be a continuous random variable with  $P(0 \leq X \leq c) = 1$  where  $c$  is a real number and  $c > 0$ . Do we have  $\text{Var}(X) \leq (c^2/4)$ ? Please give the reasons.

8. (10%) Let the joint probability density function of  $X$  and  $Y$  be given by

$$f(x, y) = \begin{cases} ce^{-x}, & \text{if } x \geq 0, |y| < x; \\ 0, & \text{otherwise} \end{cases}$$

where  $c$  is a real number. Calculate  $E(Y|X = 10)$ .