

國立成功大學

112學年度碩士班招生考試試題

編 號：161

系 所：生物醫學工程學系

科 目：工程數學

日 期：0206

節 次：第 1 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Please write the detailed solutions to the following questions.

1. (10 pts) $x^3y''' + x^2y'' - 2xy' + 2y = x^5 \ln x + \sin(\ln x)$. Find the general solution.

2. (10 pts) Find the general solution: $1 + \cot(x - y) - \cot(x - y)y' = 0$.

3. (15 pts)

(1) Find the Fourier series of $f(x)$ which has period 2

$$f(x) = \begin{cases} \frac{1}{2}x^2, & 0 < x < 2 \\ 7, & x = 0, 2 \end{cases}$$

(2) Calculate $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$

4. (10 pts) Let $A = \begin{bmatrix} 2 & 0 & 3 \\ 4 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}$, find A^n .

5. (10 pts) Find the circulation of $\vec{F} = (x - y)\vec{i} + x^2\vec{j} + xz\vec{k}$ counterclockwise around the curve enclosed by $x + 2y + z = 6$ in the first octant

6. (20 pts) Solve the partial differential equation $\nabla^2 u(x, y) = 0$ for $0 < x < 2$, $0 < y < \pi$ with the boundary conditions

$$u(x, \pi) = 0, u(x, 0) = \cos \pi x \text{ for } 0 \leq x \leq 2$$

$$u(0, y) = 0, u(2, y) = \cos y \text{ for } 0 \leq y \leq \pi$$

7. (25 pts) Find the series solution for $x^2y'' + xy' + x^2y = 0$ around $x = 0$.