系所別：電腦與通訊工程系　　組別：晶片設計組
考科代碼：1215　　考科：線性代數

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注意事項：
1、**本科目得使用本校提供之電子計算器。**
2、請於答案卷上規定之範圍作答，違者該題不予計分。

請依題目順序作答，並寫出主要的推導或計算過程（否則將扣分）。

1 Let $A\mathbf{x} = \mathbf{b}$ be a linear system whose augmented matrix $(A \mid \mathbf{b})$ has reduced row echelon form $\left[\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & 7 & -3 \\ 0 & 0 & 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$. If $\mathbf{a}_1 = \begin{bmatrix} 5 \\ 0 \\ 1 \\ 3 \end{bmatrix}$ and $\mathbf{a}_4 = \begin{bmatrix} 7 \\ 3 \\ 0 \\ 5 \end{bmatrix}$ are the first and the fourth column vectors of A, respectively, determine $\mathbf{b}$. (If $\mathbf{b}$ can not be determined, answer **NOT EXIST**) (10%)

2 Given $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 6 \\ -3 & 3 & -5 \end{bmatrix}$ and $\mathrm{b} = \begin{bmatrix} 9 \\ 0 \\ \beta \end{bmatrix}$, where β is a real number.

2.1 For what values of β will the system $A\mathbf{x} = \mathbf{b}$ have infinite many solutions ? (If no such β exists, answer **NOT EXIST**) (4%)

2.2 Find matrices E_1, E_2 and U such that $E_2 E_1 A = U$, where E_1 and E_2 are elementary (not identity) matrices, and U is a unit upper triangular matrix. (6%)

3 Evaluate the determinant of $\begin{bmatrix} e & 1+e & 2+e & 3+e \\ -1+2e & 2e & 1+2e & 2+2e \\ -2+3e & -1+3e & 3e & 1+3e \\ -3+4e & -2+4e & -1+4e & 4e \end{bmatrix}$, where $e =$ 2.7183. (10%)

4 Given $\mathbf{x}=\begin{bmatrix}5\\9\end{bmatrix}$, $\mathbf{v}_1=\begin{bmatrix}1\\2\end{bmatrix}$, $\mathbf{v}_2=\begin{bmatrix}1\\3\end{bmatrix}$, and $S=\begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix}$. Find the coordinate vector of $\mathbf{x}$ with respect to $[\mathbf{u}_1, \mathbf{u}_2]$, where S will be the transition matrix from $[\mathbf{u}_1, \mathbf{u}_2]$ to $[\mathbf{v}_1, \mathbf{v}_2]$. (10%)

5 Let $L : R^2 \rightarrow R^3$ be a linear transformation defined by $L(\mathbf{x}) = (x_1, x_2, 2x_1 - 3x_2)^T$, where $\mathbf{x} = (x_1, x_2)^T$.

5.1 Is L one-to-one ? Please answer **Yes** or **No**. (5%)

5.2 Determine the image of the subspace S, which is spanned by $(1, 0)^T$. (5%)

5.3 Find the matrix A representing L with respect to $[\mathbf{u}_1, \mathbf{u}_2]$ and $[\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$, where $\mathbf{u}_1=(5, 3)^T$, $\mathbf{u}_2=(4, 1)^T$, $\mathbf{b}_1=(1, 0, 1)^T$, $\mathbf{b}_2=(0, 1, 0)^T$, $\mathbf{b}_3=(1, 1, 2)^T$. (10%)

6 Given $A=\begin{bmatrix}1 & -1 & 4\\1 & 4 & -2\\1 & 4 & 2\\1 & -1 & 0\end{bmatrix}$.

6.1 Factor $A = QR$, where Q is an 4×3 matrix with orthonormal column vectors and R is an upper triangular 3×3 matrix whose diagonal entries are all positive. (10%)

6.2 Find a vector $\mathbf{q}_4 \in R^4$ such that the augmented matrix $(Q \mid \mathbf{q}_4)$ is an orthogonal matrix. (10%)

7 Let $A=\begin{bmatrix}2 & 2 & -1\\2 & 5 & -2\\3 & 6 & -2\end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix}8\\0\\6\end{bmatrix}$. (Hint: $3^{10}=59049$)

7.1 Find a nonsingular matrix X and a diagonal matrix D such that A can be factored into a product $A = XD^2X^{-1}$. (10%)

7.2 Compute $A^{10}\mathbf{b}$ (10%)