

國立高雄第一科技大學 100 學年度 碩士班 招生考試 試題紙

系所別：電腦與通訊工程系

組別：電腦組

考科代碼：1213

考科：離散數學

注意事項：

- 1、本科目得使用本校提供之電子計算器。
- 2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. Let the function $f: Z \rightarrow N$ be defined by

$$f(x) = \begin{cases} kx - 1, & \text{if } x > 0 \\ -kx, & \text{for } x \leq 0 \end{cases}$$

where k is a positive integer greater than 1.

(a) Prove that $f(x)$ is one-to-one using the fact that if $f(a_1) = f(a_2)$

implies $a_1 = a_2$ for all $a_1, a_2 \in Z$. (10%)

(b) Determine the inverse function of $f(x)$, $f^{-1}(x)$. (10%)

2. Let $A = \{1, 3, 4, 6, 9\}$ and R be the relation on A . xRy represents the relation that x is a multiple of y , where $x, y \in A$.

(a) Represent R as a set of ordered pairs, a Boolean matrix, and a directed graph, respectively. (10%)

(b) Represent R^2 using a set of ordered pairs, a Boolean matrix, and a directed graph, respectively. (10%)

3. In how many ways can the 26 letters of the alphabet, a, b, c, ..., x, y, z, be permuted so that none of the patterns *spin*, *nets*, *cade*, or *eat* occurs? (10%)

4. Let $f(x)$ be the generating function for the sequence: a_0, a_1, a_2, \dots

For what sequence is $\frac{(1+x)f(x)}{x-1}$ the generating function? (10%)

5. For $n \geq 4$, let $G_n = (V, E)$ be the undirected graph obtained from the complete graph K_n upon deletion of two edges connected to the same vertex. Determine the chromatic number of G_n , $\chi(G_n)$, and the chromatic polynomial of G_n using at most λ colors, $P(G_n, \lambda)$. (10%)
6. Let $T = (V, E)$ be a complete n -ary tree of height h . If T is balanced, but not full, let m denote the number of leaves in T .
- (a) Determine the minimum and maximum values for m . (10%)
 - (b) Determine the minimum and maximum values for $|E|$. (10%)
7. Given $(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$, where $x, y \in \mathbb{Z}$ and n is a prime, prove that $(x+y)^n \equiv x^n + y^n \pmod{n}$. (10%)