國立成功大學

112學年度碩士班招生考試試題

編　號：73

系　所：機械工程學系

科　目：工程數學

日　期：0206

節　次：第 3 節

備　註：不可使用計算機

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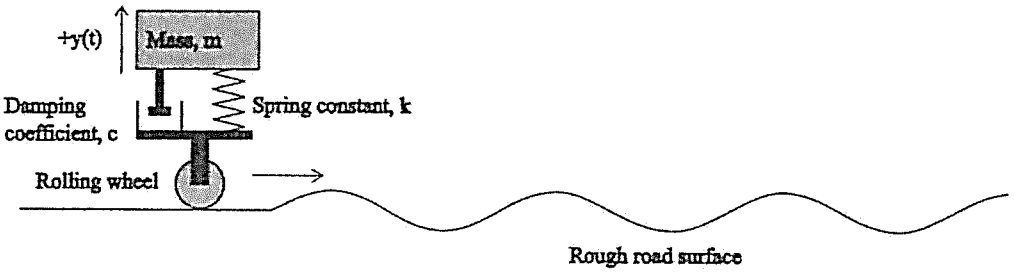
※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1) The suspension system of a motorcycle can be modeled by a spring-damp system, in which $m = 120\text{kg}, c = 1920\text{N-s/m}, k = 1200\text{N/m}$. The vehicle rolling over the road surface can be described by $r(x) = 1200sin(10t)$. The system can be modeled by $my''(t) + cy'(t) + ky(t) = r(x);\ y(0) = 0;\ y'(1) = 5m/s$ (20 pts)



my"(t) + cy'(t) + ky(t) = r(x); y(0) = 0; y'(1) = 5m/s

(a) Solve the differential equation with $c = 0$ N-s/m (8pts)

(b) Solve the differential equation with $c = 1920$ N-s/m. (8pts)

(c) Which of the above c may result in immediate and unexpected structural failure (No points will be given if the explanation is incorrect) (4pts)

2) $\begin{cases} y_1' &= & -y_1 + y_2 \\ y_2' &= & y_1 - 2y_2 + y_3 \\ y_3' &= & y_2 - y_3 \end{cases}$ with $y_1(0) = 0;\ y_2(0) = 1;\ y_3(0) = 0$, what are the limits of $y_1(t), y_2(t)\ and\ y_3(t)$ as $t \to \infty$ (15 pts)

3) The contour map of a function $f(x,y) = (\frac{3}{4}x - \frac{3}{2})^2 + (y-2)^2 + \frac{1}{4}xy$ is represented by the following figure. Given that the initial position is x=5, y=4, find the unit vector of steepest descent direction, i.e. the direction of maximum decrease. After moving one step of the unit vector, what is location of the next position (x_1, y_1)? What is the value of $f(x_1, y_1)$? (15pts)

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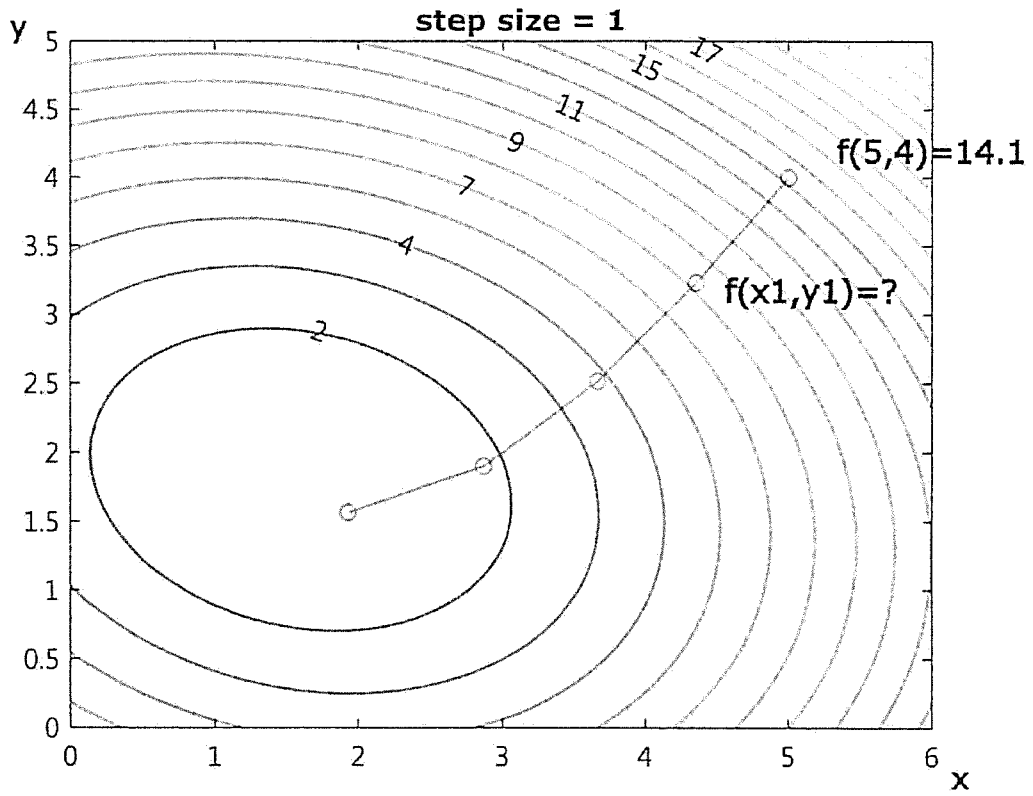

4. Please use the Laplace transform to solve the following integral equation. (10pts)

$$f(t) = 1 + t - \frac{8}{3}\int_0^t (\tau - t)^3 f(\tau)\, d\tau$$

5. (a) Find the Fourier transform of following equation (8pts)

$$f(t) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

(b) To use the result to evaluate (7pts)

$$\int_{-\infty}^{\infty} \frac{\sin w \cos wx}{w}\, dw$$

6. To evaluate (a) $\oint_c \frac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)(z-2)} dz$; (b) $\oint_c \frac{e^{2z}}{(z-1)^4} dz$, where C is $|z-1| = 2$ (10pts)

7. Please find $\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial \phi}{\partial t}$, $0 < x < \infty$, $t > 0$, and (15pts)

(1) Boundary conditions are $\phi(0, t) = 0$, $\phi(\infty, t) =$ bounded

(2) Initial condition $\phi(x, 0) = f(x)$