國立成功大學

112學年度碩士班招生考試試題

編　號：148

系　所：環境工程學系

科　目：工程數學

日　期：0206

節　次：第 3 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please solve the following differential equations: (5 points for each one)

A. $\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2+1=0$ B. $x^2\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=x^4e^x$

C. $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=e^x$ D. $\begin{cases}\dfrac{dx}{dt}+2x+6\displaystyle\int_0^t y\,d\tau=-2\\ \dfrac{dx}{dt}+\dfrac{dy}{dt}+y=0\end{cases}$ with $\begin{cases}x(0)=-5\\ y(0)=6\end{cases}$

2. Second order Runge-Kutta method is used for the first-order differential equation $\dfrac{dy}{dx}=f(x,y)$, please derive the truncation error. (15 points)

3. Finite difference method is used for the differential equation $y''+p(x)y'+q(x)y=r(x)$ with boundary conditions $y(0)=2$ and $y'(1)=0$, please derive the matrices A and B for AY=B,

where $Y=\begin{bmatrix}y_1\\ y_2\\ y_3\\ y_4\end{bmatrix}$ and y_1, y_2, y_3 and y_4 are values of y at x is 0.25, 0.5, 0.75, and 1.0,

respectively. (15 points)

4. For differential equation $\dfrac{\partial^2u}{\partial x^2}+\dfrac{\partial^2u}{\partial y^2}=0$, please find the solutions for the following conditions.

(15 points for each one)

A. $\begin{cases}u(0,y)=1, \quad \lim\limits_{x\to\infty}u(x,y)=0, \quad 0<y<1\\ \left.\dfrac{\partial u}{\partial y}\right|_{y=0}=0, \left.\dfrac{\partial u}{\partial y}\right|_{y=1}=-u(x,1), \quad x>0\end{cases}$ B. $\begin{cases}u(0,y)=0, \quad u(\pi,y)=e^{-y}, \quad y>0\\ \left.\dfrac{\partial u}{\partial y}\right|_{y=0}=0, \quad 0<x<\pi\end{cases}$

5.Gaussian plume dispersion can be derived by the differential equation $u\dfrac{\partial C}{\partial x}=k\dfrac{\partial^2C}{\partial y^2}$ where C, u, x, y and k are concentration, wind speed, coordinates x and y and diffusivity,

respectively. The concentrations are given as $C(0,y)=C_o$, $\left.\dfrac{\partial C}{\partial y}\right|_{y=0}=0$ and $C(x,\infty)=0$.(20

points)