國立高雄第一科技大學 100 學年度 碩士班 招生考試　試題紙

系 所 別：光電工程研究所　　　　組　別：不分組
考科代碼：2222　　　　　　　　　考　科：工程數學 II

注意事項：
1、本科目得使用本校提供之電子計算器。
2、請於答案卷上規定之範圍作答，違者該題不予計分。

1. (10%) Find a unit vector perpendicular to the plane $4x-2y+4z=-5$ as well as the distance of the plane from the origin.

2. (10%) Find the inverse $\mathbf{A}^{-1}$ of

$$\mathbf{A}=\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}.$$

3. (10%) Find the eigenvalues and eigenvectors of the matrix

$$\mathbf{A}=\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}.$$

4. (10%) Find out what type of conic section the following quadratic form represents and transform it to principal axes:

$$7x_1^2+48x_1x_2-7x_2^2=25.$$

Express $\mathbf{x}^{\mathrm{T}}=[x_1 \quad x_2]$ in terms of the new coordinate vector $\mathbf{y}^{\mathrm{T}}=[y_1 \quad y_2]$.

5. (10%) Find a unit normal vector $\mathbf{n}$ of the cone of revolution $z^2=4(x^2+y^2)$ at the point $P:(1,0,2)$.

6. (10%) Evaluate $\int_C \mathbf{F}\cdot\mathbf{r}'ds$, where C is the circle $x^2+y^2=4$, $z=-3$, oriented counterclockwise as seen by a person standing at the origin, $\mathbf{r}'=d\mathbf{r}/ds$ is the unit tangent vector, s is the arc length of C, and, with respect to right-handed Cartesian coordinates,

$$\mathbf{F}=y\hat{i}+xz^3\hat{j}-zy^3\hat{k}.$$

7. (10%) Evaluate

$$I=\iint_S (x^3dydz+x^2ydzdx+x^2zdxdy)$$

where S is the closed surface consisting of the cylinder $x^2+y^2=a^2$ $(0\le z\le b)$ and the circular disks $z=0$ and $z=b$ ($x^2+y^2\le a^2$).

8. (10%) Find the solution $u(x, y)$ of the following equation:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0.$$

9. (10%) The vibrations of an elastic string are governed by the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2},$$

where $u(x, t)$ is the deflection of the string. Find the deflection $u(x, t)$ of the vibrating string (length $L=\pi$, ends fixed, and $c^2=1$) corresponding to zero initial velocity and initial deflection $k\sin 2x$.

10. (10%) Show that $u(x,t)=f(x+ct)+g(x-ct)$ is a solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2};$$

here, f and g are any twice differentiable functions.