國立成功大學

112學年度碩士班招生考試試題

編　號：191

系　所：電腦與通信工程研究所

科　目：機率與線性代數

日　期：0206

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) Let $X_1, X_2, \ldots$ be a sequence of independently and identically distributed random variables, each with expectation μ and variance σ^2, prove that the distribution of

$$Z_n = \frac{X_1 + X_2 + \cdots + X_n - n\mu}{\sigma\sqrt{n}}$$

converges to the distribution of a standard normal distribution as $n \to \infty$.

2. (15%) Let X and Y be independent standard normal random variables (zero mean, unit variance). Find the joint pdf of V and W defined by $V = \sqrt{X^2 + Y^2}$ and $W = \angle(X, Y)$, where $\angle\theta$ denotes the angle in the range $(0, 2\pi)$ that is defined by the point (x, y).

3. (10%) Let X be a random variable with pdf, $f_X(x) = e^{-x}$, $x > 0$. Find the pdf of $Y = 1/X^2$, $Z = 1/X^3$.

4. (10%) What is the probability of an odd number of successes in n independent Bernoulli trials ? (Bernoulli success probability is p)

5. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)

   (a) Let A be an invertible matrix. Then $I + A$ is also an invertible matrix, where I is the identity matrix of the same size as A.

   (b) If A is a real-valued square matrix of size n, then $\det(I + A^T A) > 0$, where I is the identity matrix of the same size as A.

   (c) For a square matrix M, we have $\text{rank}(M^2) = \text{rank}(M)$.

   (d) Suppose three $n \times n$ matrices A, B, and C satisfy $AB = AC$, where A is invertible. Then we have $B = C$.

6. (20%) Consider a linear transformation T on $\mathbb{R}^3$, define by

$$T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a+c \\ b+a \\ c+b \end{bmatrix}$$

Find the standard matrix of T. Also, find the inverse of T. (Express your answer as $T^{-1}\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \cdots$)

7. (10%) Suppose the characteristic polynomial of a 4×4 matrix M is $p(t) = t^4 - t^3 + 5t^2 - 3t + 7$. Explain why M is invertible.