國立成功大學

112學年度碩士班招生考試試題

編　號：185

系　所：電腦與通信工程研究所

科　目：電磁數學

日　期：0206

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Evaluate the integral $\oint_C \frac{5z+7}{z^2+2z-3}$, $\mathbf{C}$: the circle $|z-2|=2$ counterclockwise.

2. (20%) Solve $2xy^2+yy''=(y')^2$, $y(0)=1$, $y'(0)=2$.

3. (20%) Solve the ordinary differential equation in terms of Bessel function,

$$4x^2y''-20xy'+(4x^2+35)y=0.$$

4. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)

   (a) Let A be an invertible matrix. Then $I+A$ is also an invertible matrix, where I is the identity matrix of the same size as A.

   (b) If A is a real-valued square matrix of size n, then $\det(I+A^TA)>0$, where I is the identity matrix of the same size as A.

   (c) For a square matrix M, we have $\text{rank}(M^2)=\text{rank}(M)$.

   (d) Suppose three $n\times n$ matrices A, B, and C satisfy $AB=AC$, where A is invertible. Then we have $B=C$.

5. (20%) Consider a linear transformation T on $\mathbb{R}^3$, define by

$$T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right)=\begin{bmatrix} a+c \\ b+a \\ c+b \end{bmatrix}$$

   Find the standard matrix of T. Also, find the inverse of T. (Express your answer as $T^{-1}\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right)=\cdots$)

6. (10%) Suppose the characteristic polynomial of a 4×4 matrix M is $p(t)=t^4-t^3+5t^2-3t+7$. Explain why M is invertible.