

國立成功大學

112學年度碩士班招生考試試題

編 號：237

系 所：工業與資訊管理學系

科 目：統計學

日 期：0207

節 次：第 3 節

備 註：可使用計算機

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

I. (50%, 5% each)

- I-1.() Which of the following statements is correct when events A and B are exclusive events? Notice that $\Pr(\cdot)$ is the probability of an event occurring, $\Pr(A) \neq 0$ and $\Pr(B) \neq 0$.
- (A) $\Pr(A \cup B) = \Pr(A)$.
 (B) $\Pr(A|B) = \Pr(A)$.
 (C) $\Pr(A|B) = \Pr(B)$.
 (D) $\Pr(B) \leq \Pr(A^c)$
 (E) Events A and B are independent.
- I-2.() It is known $B_1 \cup B_2 \cup B_3 = \text{Sample space}$ and they are mutually exclusive. In addition, $\Pr(B_1) = 0.3$, $\Pr(B_2) = 0.5$, and $\Pr(B_3) = 0.2$. Moreover, given $\Pr(A|B_1) = 0.1$, $\Pr(A|B_2) = 0.2$, and $\Pr(A|B_3) = 0.3$, what is the value of $\Pr(A)$?
- (A) 0.22 (B) 0.19 (C) 0.35 (D) 0.43 (E) 0.56.
- I-3.() A random variable X has cumulative distribution function $F(x) = 1 - (0.2)^{x+1}$, $x = 0, 1, 2, \dots$. What is the expected value of X ?
- (A) 0.2 (B) 0.8 (C) 0.16 (D) 0.25 (E) 4.
- I-4.() A random variable Y has cumulative distribution function $F(y) = 1 - e^{-y/20}$, $y > 0$. What is the expected value of Y ?
- (A) 0 (B) 1/20 (C) 20 (D) 40 (E) 120.
- I-5.() Let X be another random variable such that $E[X|Y=y] = 10 \times y$ where the random variable Y has CDF $F(y) = 1 - e^{-y/20}$, $y > 0$. What is the expected value of X , $E[X]$?
- (A) 200 (B) 20 (C) 10 (D) 1/20 (E) 1/10.
- I-6.() A random variable X follows a binomial distribution $B(n, p)$. Under what condition does the inequality $\left| \Pr(X \leq 0) - \Phi\left(-\frac{np}{\sqrt{np(1-p)}}\right) \right| \leq 10^{-3}$, where $\Phi(\cdot)$ is the CDF of standard normal distribution, hold?
- (A) $(n, p) = (2, 0.1)$ (B) $(n, p) = (3, 0.1)$ (C) $(n, p) = (5, 0.1)$ (D) $(n, p) = (10, 0.2)$ (E) $(n, p) = (50, 0.2)$.
- I-7.() It is known that the lifetime of a particular type of electronic device follows an exponential distribution (PDF: $f(x) = 1/\theta \times e^{-x/\theta}$ where $x > 0$ and $\theta > 0$) with mean 30,000 hours. The company provides a one-year free-replacement warranty policy if a sold device fails in a year. What is the probability that a sold device fails in one year (i.e., $\Pr(X \leq 8,760)$)?
- (A) 55% (B) 44% (C) 25% (D) 36% (E) 100%.

- I-8.() Following the previous question (see question I-7), there were 1,000 electrical devices sold last year. On average, how many devices may fail and return for a free replacement in a year?
(A) 253 (B) 446 (C) 360 (D) 152 (E) 875.
- I-9.() When two random variables X and Y have zero correlation coefficient, it means:
(A) The two random variables are identical, $X = Y$.
(B) The two random variables satisfy $X = a \times Y + b$ where a, b are two non-zero constants.
(C) The two random variables are independent.
(D) One of the two random variables has zero variance.
(E) The two random variables have zero covariance.
- I-10.() Which of the following statement about central limit theorem is correct (x_1, \dots, x_n is a selected random sample)?
(A) The theorem can be applied when the sample size is small (e.g., $n = 2$).
(B) The theorem suggests that when the sample size is large, the distribution of x_i becomes a normal distribution.
(C) The sampling distribution of sample mean \bar{x} is a normal distribution.
(D) The sampling distribution of sample mean \bar{x} must be a standard normal distribution.
(E) The sampling distribution of sample variance S^2 is a normal distribution.

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II-1. (17%) The response values obtained from Group A are (12, 13, 54, 56, 25).

Please perform the Jarque-Bera test to determine whether the sample data matches a normal distribution.

II-2. (8%) The response values obtained from Group B are (43, 48, 6, 62, 56).

Assuming that the data is normally distributed, please perform the Grubbs' test to determine whether there is an outlier in the dataset.

II-3. (25%) The following data points are obtained from a randomized complete block design. In here, we assume that all assumptions required for the ANOVA analysis are fulfilled. The level of significance was set at 0.05. Please answer the following questions:

Group	1	2	3	4	5
A	12	13	54	56	25
B	43	48	6	62	56
C	49	81	64	78	89

(1). (13%) The F-value for the group effect. What can you conclude from that value?

(2). (5%) The eta squared for the group effect. Please interpret its meaning.

(3). (7%) Please use the Bonferroni correction for post hoc tests to estimate the 95% confidence interval for the mean difference between Group B and Group C.

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of χ^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89

Table: The critical values of G for the corresponding alpha levels.

N	Alpha				
	0.1	0.075	0.05	0.025	0.01
3	1.15	1.15	1.15	1.15	1.15
4	1.42	1.44	1.46	1.48	1.49
5	1.6	1.64	1.67	1.71	1.75
6	1.73	1.77	1.82	1.89	1.94
7	1.83	1.88	1.94	2.02	2.1
8	1.91	1.96	2.03	2.13	2.22
9	1.98	2.04	2.11	2.21	2.32
10	2.03	2.1	2.18	2.29	2.41
11	2.09	2.14	2.23	2.36	2.48
12	2.13	2.2	2.29	2.41	2.55
13	2.17	2.24	2.33	2.46	2.61
14	2.21	2.28	2.37	2.51	2.66
15	2.25	2.32	2.41	2.55	2.71
16	2.28	2.35	2.44	2.59	2.75
17	2.31	2.38	2.47	2.62	2.79

IV Percentage Points of the F Distribution (Continued)

F_{α, ν_1, ν_2}

ν_1	Degrees of Freedom for the Numerator (ν_1)																	∞	
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60		120
2	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
3	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
4	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
5	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
6	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
7	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
8	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
9	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.43	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
10	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
11	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
12	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
13	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
14	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
15	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
16	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
17	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
18	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
19	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
20	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
21	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
22	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
23	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
24	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
25	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
26	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
27	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
28	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
29	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
30	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
40	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
60	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Degrees of Freedom for the Denominator (ν_2)

II Percentage Points of the *t* Distribution^a

$\nu \backslash \alpha$	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.727	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.019	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

ν = Degrees of freedom.

^aAdapted with permission from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition, by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966.