國立成功大學
112學年度碩士班招生考試試題

編　號：233

系　所：統計學系

科　目：數理統計

日　期：0207

節　次：第 2 節

備　註：不可使用計算機

編號： 233

國立成功大學 112 學年度碩士班招生考試試題

系　所：統計學系

考試科目：數理統計

考試日期：0207，節次：2

第1頁，共1頁

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分

1. Let $X_1, \ldots, X_n$ be a sample from a population with the Rayleigh density

$$f(x;\theta) = \frac{x}{\theta^2} \exp\left\{-\frac{x^2}{2\theta^2}\right\},\ x, \theta > 0.$$

(a) (5%) Construct a level α hypothesis test of $H_0:\ \theta = 1$ versus $H_1:\ \theta > 1$.

(b) (10%) Find the 95% confidence interval of θ.

2. (15%, 5% for each) Let $X_1, \ldots, X_n$, $n \geq 2$, be independently and identically distributed with density

$$f(x;\theta) = \frac{1}{\sigma} \exp\{-(x-\mu)/\sigma\},\ x \geq \mu,$$

where $\theta = (\mu, \sigma)$, $-\infty < \mu < \infty$, $\sigma > 0$.

(a) Find a method of moment estimator for θ.

(b) Find the maximum likelihood estimator of θ.

(c) Find the maximum likelihood estimator of $P_\theta[X_1 \geq t]$ for $t \geq \mu$.

3. (15%) Let X has a continuous uniform distribution on the interval $(0, 2\pi)$. Consider $Y = \sin^2(X)$. Find the cumulative density function (cdf) of Y.

4. (20%, 10% for each) Suppose $X_1, X_2, \ldots$ are jointly continuous and independent, each distributed with marginal pdf $f(x)$, where each X_i represents annual rainfall at a given location.

(a) Find the distribution of the number of years until the first year's rainfall, X_1, is exceeded for the first time.

(b) Show that the mean number of years until X_1 is exceeded for the first time is infinite.

5. (20%, 10% for each) Suppose that $X_1, \ldots, X_n|\theta \overset{iid}{\sim} N(\theta, \sigma^2)$ where $\sigma > 0$ is known. Suppose $\theta \sim \mathcal{DE}(\lambda)$ where

$$\pi(\theta) = \frac{\lambda}{2} e^{-\lambda|\theta|},\ \theta \in \mathbb{R},\ \lambda > 0.$$

(a) Find the posterior distribution of θ given $X_1, \ldots, X_n$.

(b) Find a level $1-\alpha$ credible region for θ given $X_1, \ldots, X_n$.

6. (15%) Let X be any random variable, and $g(x)$ and $h(x)$ be any functions such that all of the $E[g(X)], E[h(X)]$, and $E[g(X)h(X)]$ exist. If assuming $g(x)$ is a nondecreasing function and $h(x)$ is a nonincreasing function, then prove that

$$E[g(X)h(X)] \leq E[g(X)]E[h(X)].$$