國立成功大學

112學年度碩士班招生考試試題

編　號：234

系　所：統計學系

科　目：統計學

日　期：0207

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

A. True or False (Please answer "T" for true and "F" for false statement, 10 x 2% = 20%)

1. The events A and B have nonzero probabilities, A and B can be both independent and mutually exclusive.
2. In point estimation, data from the sample is used to estimate the population parameter.
3. For a data set, half of the observations are always greater than the mean.
4. Bayes' Theorem is used to compute posterior probabilities.
5. If the outcomes of a random variable follow a Poisson distribution, then their mean equals the standard deviation.
6. The probability of "rejecting a null hypothesis when the null hypothesis is true" is known as the p-value.
7. It is appropriate to use the paired difference t-test, when two independent samples are compared.
8. Analysis of variance (ANOVA) is used to simultaneously compare several population means.
9. For a continuous random variable X, the height of the function at $X=x$ is named the probability mass function $f(x)$.
10. For a given sample size in hypothesis testing, the smaller the Type I error, the larger the Type II error will be.

B. Multiple Choice (15 x 3% = 45%)

1. Assume that the height of men is normally distributed with a mean of 1.85 meter and variance of 0.09. What is the probability that the height of a man is between 1.65 and 1.95 meters?
   (A) 0.0132　(B) 0.3779　(C) 0.4596　(D) 0.5000　(E) 0.8533
2. A random sample of 36 persons was conducted. The sample mean for the annual premium for automobile insurance in the United States is 2300. The sample standard deviation is 120. Let μ be the mean annual automobile insurance premium in the United State. Develop a 99% confidence interval for μ.
   (A) [2245.52, 2354.48]　(B) [2251.24, 2348.76]　(C) [2245.62, 2354.38]
   (D) [2248.48, 2351.52]　(E) [2260.80, 2339.20]
3. Consider the experiment of rolling a pair of dice. Suppose that we are interested in the sum of the face values showing on the dice. What is the probability of obtaining a value of 9 or greater?
   (A) 3/36　(B) 6/36　(C) 10/36　(D) 15/36　(E) 21/36
4. Which of the following are continuous random variables?
   I. The weight of an elephant
   II. The time to answer a questionnaire
   III. The number of floors in a skyscraper
   IV. The square feet of countertop in a kitchen
   (A) I and II only　(B) III and IV only　(C) I, II and IV only　(D) I, II, III, and IV　(E) None of above

Exhibit 1: (Questions 5~6)

The company has been basing its premium on an assumption that the average claim is $1200. You want to raise the premium, and a regulator has insisted that you provide evidence that the average now exceeds $1200. Suppose you have a sample of 49 claims with sample mean is $1215. Use a population variance of $1225 claims and 1% level of significance.

5. Refer to **Exhibit 1.** Formulate hypotheses that can be used to test the rationality of the premium arisen. What is the critical value for the hypothesis?

(A) $H_0: \mu \leq 1200, H_a: \mu > 1200, 2.326$

(B) $H_0: \mu < 1200, \ H_a: \mu \geq 1200, 2.326$

(C) $H_0: \mu \geq 1200, H_a: \mu < 1200, -2.326$

(D) $H_0: \ \mu > 1200, \ H_a: \mu \leq 1200, -2.326$

(E) $H_0: \mu = 1200, \ H_a: \mu \neq 1200, \pm 2.576$

6. Refer to **Exhibit 1.** What is the p-value for this test, and what is your decision?

(A) 0.2743, reject H_0

(B) 0.2743, do not reject H_0

(C) 0.0026, reject H_0

(D) 0.0013, reject H_0

(E) 0.0013, do not reject H_0

Exhibit 2. (Questions 7~8)

In a completely randomized design, 12 experimental units were used for the first treatment, 15 for the second treatment, and 20 for the third treatment. Complete the following analysis of variance.

Source	Sum of Squares (SS)	Degrees of Freedom (DF)	Mean Squares (MS)	F
Treatments	1200	?	?	?
Error	?	?	?	
Total	1800	?		

7. Refer to **Exhibit 2.** What are the mean square error (MSE) and F-statistic?

(A) MSE=13.64, F=42.00

(B) MSE=13.64, F=43.99

(C) MSE=600, F=43.99

(D) MSE=1200, F=43.99

(E) None of above

8. Refer to **Exhibit 2.** If sum of squares due to treatments (SSTR) changes from 1200 to 1000, what can you say about the F-statistic and the p-value?

(A) both of them decrease

(B) both of them increase

(C) F increases, p-value decreases

(D) F deceases, p-value increase

(E) Nothing happens

Exhibit 3. (Questions 9~10)

An automobile dealer conducted a test to determine if the time in minutes needed to complete a minor engine tune-up depends on whether a computerized engine analyzer or an electronic analyzer is used. Because tune-up time varies among compact, intermediate, and full-sized cars, the three types of cars were used as blocks in the experiment. The data are given below.

		Analyzer		
		Computerized	Electronic	Row means $\bar{x}_{i\cdot}$
Car	Compact	30	22	26
	Intermediate	35	24	29.5
	Full-sized	43	26	34.5
Column means $\bar{x}_{\cdot j}$		36	24	overall mean $\bar{\bar{x}} = 30$

9. Refer to **Exhibit 3.** What is the value of sum of squares due to blocks (SSBL)?

(A) 34 (B) 36.5 (C) 68 (D) 73 (E) 216

10. Refer to **Exhibit 3.** Given the total sum of squares (SST= 310), and sum of squares due to treatments (SSTR=216), what is the value of the sum of squares due to error (SSE) and its corresponding degrees of freedom (DF)?

(A) SSE=10.5; DF =2 (B) SSE=10.5; DF =1 (C) SSE=21; DF =2

(D) SSE=21; DF =1 (E) SSE=94; DF =2

Exhibit 4. (Questions 11~12)

A regression analysis of the relationship between X, number of salespersons at a branch office, and Y, annual sales at the office (in thousands of dollars) provided the following computer output (with some missing values) from a regression analysis of the data.

The regression equation is

Y = 66.1 + 0.402 X

Predictor	Coef	SE Coef	T	p-value
Constant	66.10	32.06	2.06	0.094
X	0.402	0.2276	1.77	0.137
s = 12.62	R-sq = ?	R-sq(adj) =?		

Note: Coef - Regression Coefficient; SE Coef - Standard Error of the Coefficient

Analysis of Variance

SOURCE	DF	SS	MS	F	p-value
Regression		497.2			
Residual Error	5	795.7			
Total	6	1292.9			

Note: DF - Degrees of Freedom; SS - Sum of Squares; MS - Mean Squares

11. Refer to **Exhibit 4.** What is the value of F statistic?

(A) between 0 and 1 (B) between 1 and 2 (C) between 2 and 3 (D) larger than 3 (E) larger than 4

12. Refer to **Exhibit 4.** What is the adjusted R-square value?
(A) 0.261 (B) 0.385 (C) 0.539 (D) 0.615 (E) 0.912

13. If X and Y are independent random variables, which of the following identities is false?
(A) $P(X = x, Y = y) = P(X = x)P(Y = y)$
(B) $\mathrm{Cov}(X, Y) = 0$
(C) $E(X + Y) = E(X) + E(Y)$
(D) $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$
(E) $\mathrm{Cov}(X, Y) = 1$

14. For a chi-square goodness-of-fit-test, the calculated chi-square value is 7.21. If the table chi-square value is 10.645, what is the appropriate decision for this test?
(A) Reject the null hypothesis
(B) Fail to reject the null hypothesis
(C) Reject the alternative hypothesis
(D) Fail to reject the alternative hypothesis
(E) It is impossible to determine anything from the given information

15. Consider the following hypothesis test: $H_0: \mu = 18$ versus $H_a: \mu \neq 18$. A sample of 100 provided a sample mean 17.5 and a sample standard deviation 2.8. Using a 0.05 level of significance, what is the test statistic? What is the conclusion for the hypothesis test?
(A) ± 1.984; reject the null hypothesis
(B) ± 1.984; do not reject the null hypothesis
(C) ± 1.660; do not reject the null hypothesis
(D) ± 1.786; reject the null hypothesis
(E) ± 1.786; do not reject the null hypothesis

C. Problems (35%)

1. Consider a simple linear regression analysis. Let x_i be the independent variable and y_i be the response variable, for $i = 1, \ldots, n$. Please answer the following questions:

(1) Write down the simple linear regression model and state the basic model assumption. (5%)

(2) Let $\bar{x}$ and $\bar{y}$ be the sample means for the independent and dependent variables, respectively. Does the estimated (fitted) regression line pass through the point $(\bar{x}, \bar{y})$? Explain. (5%)

(3) Show that $r_{xy} = (\text{sign of } \hat{\beta}_1)\sqrt{R^2}$, where r_{xy} is the sample correlation coefficient, R^2 is the coefficient of determination, and $\hat{\beta}_1$ is the estimate of regression slope. (5%)

2. Describe/explain probability and non-probability sampling methods from a population. (10%)

3. State/describe the Central Limit Theorem (CLT). (10%)

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附表：Cumulative Probabilities for the Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

參考值：

$t_{0.025}(35) = 2.030$, $t_{0.01}(35) = 2.438$, $t_{0.005}(35) = 2.724$, $t_{0.01}(36) = 2.434$, $t_{0.005}(36) = 2.719$,
$t_{0.05}(99) = 1.660$, $t_{0.025}(99) = 1.984$, $t_{0.01}(99) = 2.364$, $t_{0.05}(100) = 1.660$, $t_{0.025}(100) = 1.984$