國立成功大學
112學年度碩士班招生考試試題

編　號：290

系　所：環境醫學研究所

科　目：微積分

日　期：0207

節　次：第 3 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15 %) Evaluate the following limits:

   (a) $\lim_{x\to+\infty}\left(\frac{x^2-4}{x^2-1}\right)^{x^2+1}$.

   (b) $\lim_{n\to+\infty}\frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n-1}}{n\sqrt{n}}$.

   (c) $\lim_{x\to1}\frac{1}{x-1}\left(\int_{1/2}^{x}\frac{1}{\sqrt{2t-t^2}}\,dt-\frac{\pi}{6}\right)$.

2. Find the following derivatives:

   (a) (10 %) Assume that the equation $\ln(x+y+z)=1+x^3y^2z^3$ defines z as a differentiable function of two independent variables x and y. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

   (b) (5 %) If $F(x)=\int_{\ln x}^{x}\frac{e^{x^2t^2}}{t}\,dt$, $x>1$, find $F'(x)$.

   (c) (5 %) Given $y=\frac{\sqrt{1+2x}\,\sqrt[4]{1+4x}\,\sqrt[6]{1+6x}\cdots\sqrt[100]{1+100x}}{\sqrt[3]{1+3x}\,\sqrt[5]{1+5x}\,\sqrt[7]{1+7x}\cdots\sqrt[101]{1+101x}}$, find y' at $x=0$.

3. Evaluate the following integrals:

   (a) (10 %) Prove the improper integral $\int_0^\infty e^{x-2e^x}\,dx$ exists and evaluate $\int_0^\infty e^{x-2e^x}\,dx$.

   (b) (5 %) If $\int_0^1(1-x)f(x)\,dx=5$, find $\int_0^1\int_0^x f(x-y)\,dydx$.

   (c) (5 %)Find the iterated integral $\int_0^{\sqrt{\frac{\pi}{2}}}\int_x^{\sqrt{\frac{\pi}{2}}}x^2\sin y^2\,dydx$.

4. Let $f(x)=\left(\int_0^x e^{-t^2}\,dt\right)^2$, $g(x)=\int_0^1\frac{e^{-x^2(t^2+1)}}{t^2+1}\,dt$.

   (a) (5 %) Prove that $g'(x)+f'(x)=0$ for any $x\in\mathbb{R}$;

   (b) (5 %) Prove that $g(x)+f(x)=\frac{\pi}{4}$ for any $x\in\mathbb{R}$;

   (c) (10 %) **Using (b)**, prove that $\int_0^{+\infty}e^{-t^2}\,dt=\frac{1}{2}\sqrt{\pi}$.

5. (15 %) Sketch the graph of $y=\tan^{-1}x-\ln\sqrt{1+x^2}$, showing its critical points, points of inflection, the intervals on which it increases or decreases, and the intervals on which the graph is concave up or concave down.

6. (10 %) Let $f(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n$, where $a_0,a_1,\ldots,a_n\in\mathbb{R}$. If $\frac{a_0}{1}+\frac{a_1}{2}+\cdots+\frac{a_n}{n+1}=0$, prove that the equation $f(x)=0$ has a root between 0 and 1.