國立交通大學 101 學年度碩士班考試入學試題

科目：應用數學(4011) 考試日期：101年2月16日 第 1 節

系所班別：電子物理學系 組別：電物系甲組 第 1 頁，共 1 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. (10%) Evaluate the line integral $\oint_C \vec{F}\cdot d\vec{r}$, where $\vec{F} = x\hat{i} - z\hat{j} + 2y\hat{k}$ and C is from (0,0,0) straight to (1,1,0), then to (1,1,1) and back to (0,0,0).

2. (15%) Evaluate the surface integral $\iint_S (x^3 dydz + x^2 ydzdx + x^2 zdxdy)$ on the surface $S: x^2 + y^2 = a^2,\ z \in [0,b]$.

3. (10%) Find the complex Fourier transform of the function $f(x) = e^{-2x^2}$.

4. (15%) Perform the integral $\text{p.v.}\int_{-\infty}^{\infty} \frac{dx}{(4x^2+1)^3}$ by contour integral method.

5. Consider the following differential equation:

$$\partial_x^2 f + \partial_y^2 f + (x+y)f = 0$$

(a) (10%) Reduce the above equation to two second-order differential equations by using separation of variables.

(b) (10%) Using Fourier transformation technique, show that each of the above second-order differential equations correspond to a first-order equation in the Fourier conjugate variable. Solve these first-order equations and then display the corresponding solution to each of the original second-order equation in a form of an integral.

(c) (10%) Collecting the result from (b) and finding an expression for the solution of f expressed as a multiple integral. Is this the most general solution?

6. Let $A = \begin{bmatrix} 10 & -5 & 7 \\ -5 & 22 & -5 \\ 7 & -5 & 10 \end{bmatrix}$

(a) (4%) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

(b) (4%) Find a matrix B such that $B^2 = A$.

(c) (4%) If Q is an orthogonal matrix and $\vec{x}$ is nonzero vector such that $Q\vec{x} = c\vec{x}$, what is c ?

(d) (4%) Following (c), show that $Q^2 = I$ if Q is orthogonal and symmetric.

(e) (4%) If Q is orthogonal and symmetric, show that $Q = \frac{1}{2}(I - Q)$ is a projection matrix.