國立政治大學 112 學年度 碩士班暨碩士在職專班 招生考試試題

考試科目	數理統計學	系所別	統計學系	考試時間	2 月 3 日(星期五) 第 2 節

- Note. All parts in the same problem are worth the same points.

1. (40 pts) Suppose that X is a random variable whose distribution is $N(\theta, 1)$, the normal distribution with mean θ and variance 1. Suppose that we have IID (independent and identically distributed) observations $Y_1, \ldots, Y_n$, and the distribution of Y_1 is the same as the distribution of X^3. Suppose that the parameter $\theta \in (-\infty, \infty)$ is unknown.

   (a) Find a PDF of X^3.

   (b) Find the maximum likelihood estimator for θ based on the observations $Y_1, \ldots, Y_n$.

   (c) For $\alpha \in (0,1)$, find the likelihood ratio test of size α for testing

   $$H_0 : \theta = 0 \text{ versus } H_1 : \theta \neq 0.$$

   Express the rejection region in terms of a test statistic and give the distribution of the test statistic under H_0.

   (d) Find the (uniformly) minimum variance unbiased estimator for θ^2 based on the observations $Y_1, \ldots, Y_n$.

2. (60 pts) Suppose that $(X_1, \ldots, X_n)$ is a random sample from the uniform distribution on $[-\theta, \theta]$, where $\theta > 0$ is an unknown parameter. Let

   $$X_{(n)} = \max_{1 \le i \le n} X_i$$

   and

   $$X_{(1)} = \min_{1 \le i \le n} X_i.$$

   (a) Show that $X_{(n)}$ is a consistent estimator for θ.

   (b) Determine whether $X_{(n)}$ and $X_{(1)}/X_{(n)}$ are independent. Justify your answer.

   (c) Find $\hat{\theta}$: an estimator for θ based on the data such that

   $$E\left((\hat{\theta} - \theta)^2\right) < E\left((X_{(n)} - \theta)^2\right)$$

   for every $\theta > 0$. Justify your answer.

   (d) Construct a 95% confidence interval for θ based on the statistic $(X_{(1)}, X_{(n)})$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。