國立政治大學　112　學年度 碩士班暨碩士在職專班 招生考試試題

考試科目	基礎數學	系所別	統計學系	考試時間	2 月 3 日(五) 第 一 節

Part I: Calculus

1. (15%) State and prove the mean value theorem.
2. (10%) Use ϵ-δ argument to show that $\lim_{x\to\infty} \frac{1}{x^2} = 0$.
3. (20%) Find the following integrals:

   (a) $\int \frac{2x^2-x+4}{x^3+4x}dx$ (b) $\int_{-2}^{3} \frac{1}{x^4}dx$ (c) $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x+\cos^n x}dx$ (d) $\int_0^{\infty} e^{-x^2}dx$ (e) $\int \sin^5 x \cos^2 x dx$.

4. (5%) Find $\frac{dy}{dx}$ if $\tan\left(\frac{x}{y}\right) = x + y$.
5. (10%) Find the extreme value of the function $f(x, y, z) = x + 2y + 3z$ subject to constraints $x - y + z = 1$ and $x^2 + y^2 = 1$.

Part II: Linear Algebra

6. (10%) Suppose that A is a matrix satisfying $A^2 = A$ and $A^\top = A$.
   (a) Find possible eigenvalues of A.
   (b) Determine the rank of A.
7. (10%) Prove that if Q is an orthogonal matrix, then $\det(Q) = 1$.
8. (15%) Let V and W be two vector spaces and let $T : V \to W$ denote a linear transformation. Suppose that $\mathcal{N}(T)$ and $\mathcal{R}(T)$ are null space and range of T, respectively. Show that $\mathcal{N}(T)$ and $\mathcal{R}(T)$ are subspaces of V and W, respectively.
9. (5%) Let $A = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$. Compute A^n for any positive integer n.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。