國立政治大學 112 學年度 碩士班暨碩士在職專班 招生考試試題

考試科目	微積分	系所別	應用數學系	考試時間	2 月 3 日 (五) 第三節

1. (8 points) Evaluate the definite integral: $\int_0^1 \frac{e^z+1}{e^z+z}dz$.

2. (10 points) Find the indefinite integral: $\int \frac{1}{(x^2+1)^2}dx$.

3. (10 points) Evaluate the following sum of integrals:

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xydydx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xydydx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xydydx.$$

4. (a) (7 points) Prove the formulation: $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx \qquad (n \neq 1).$

   (b) (7 points) Find the indefinite integral: $\int \tan^5 x dx$.

5. (10 points) Find the values of p for which the integral: $\int_0^1 x^p \ln x dx$ converges and evaluate the integral for those values of p.

6. (8 points) Use the technique of implicity function to find $\frac{\partial z}{\partial x}$ for the equation: $yz + x\ln y = z^2$.

7. (8 points) Show that the function $f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is **not** equal to its Maclaurin series.

8. (8 points) Prove that if $\lim_{n\to\infty} a_n = 0$ and $\{b_n\}$ is bounded, then $\lim_{n\to\infty} (a_n b_n) = 0$.

9. (10 points) Prove that if f is continuous, then $\int_0^x f(u)(x-u)du = \int_0^x \left(\int_0^u f(t)dt\right)du$.

10. A function f is said to be a *contraction* on $[a,b]$ if there exists K, $0 < K < 1$, such that for all $u, v \in [a,b]$, we have $|f(u) - f(v)| \le K|u-v|$.

    (a) (7 points) Show by the $\varepsilon - \delta$ definition that if f is a contraction on $[a,b]$, then f is continuous on $[a,b]$.

    (b) (7 points) Suppose that f is continuous on $[a,b]$ and differentiable on (a,b) with $|f'(x)| \le L$, $0 < L < 1$, for all $x \in (a,b)$.
    Show that f is a contraction on $[a,b]$ and have at most one fixed point on $[a,b]$.
    (A point $c \in [a,b]$ is called a fixed point if $f(c) = c$.

備 註	一. 作答於試題上者，不予計分。 二. 試題請隨卷繳交。