## 國立中正大學 111 學年度碩士班招生考試

# 試 題

### [第1節]

科目名稱	數學
系所組別	資訊工程學系-甲組、乙組

#### -作答注意事項-

- ※作答前請先核對「試題」、「試卷」與「准考證」之<u>系所組別、科目名稱</u>是否相符。
- 1. 預備鈴響時即可入場,但至考試開始鈴響前,不得翻閱試題,並不得書寫、 畫記、作答。
- 2. 考試開始鈴響時,即可開始作答;考試結束鈴響畢,應即停止作答。
- 3.入場後於考試開始 40 分鐘內不得離場。
- 4.全部答題均須在試卷(答案卷)作答區內完成。
- 5.試卷作答限用藍色或黑色筆(含鉛筆)書寫。
- 6. 試題須隨試卷繳還。



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科目名稱:數學

本科目共2頁 第1頁

系所組別:資訊工程學系-甲組、乙組

- 1. (10%) Let  $\mathbf{u} = (2, 0, 1)$  and  $\mathbf{a} = (1, 2, 3)$ . Find the vector component of  $\mathbf{u}$  along  $\mathbf{a}$  and the vector component of  $\mathbf{u}$  orthogonal to  $\mathbf{a}$ .
- 2. (10%) Find a vector that is orthogonal to both  $\mathbf{u} = (0, 2, -2)$  and  $\mathbf{v} = (1, 3, 0)$ .
- 3. (10%) Find the coordinate vector of  $\mathbf{v} = (3, 4, 3)$  relative to the basis  $S = \{(3,2,1), (-2,1,0), (5,0,0)\}$ .
- 4. (10%) Find  $A^{13}$ , where

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

5. (10%) Let the vector space  $P_2$  have the inner product

$$\langle \mathbf{p}, \mathbf{q} \rangle = \int_{-1}^{1} p(x) q(x) dx$$

Apply the Gram-Schmidt process to transform the standard basis  $\{1, x, x^2\}$  for  $P_2$  into an orthogonal basis  $\{\varphi_1(x), \varphi_2(x), \varphi_3(x)\}$ .

- 6. (10%) Show that if n is an integer and  $n^3 + 5$  is odd, then n is even using
  - a) a proof by contraposition. (5%)
  - b) a proof by contradiction. (5%)
- 7. (10%) Prove whether the given pair of graphs  $G_1$  and  $G_2$  represented by incidence matrices is isomorphic or not.

$$G_{1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \ G_{2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

8. (10%) Show that the set of positive rational numbers is countable.

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科目名稱:數學

本科目共2頁第2頁

系所組別:資訊工程學系-甲組、乙組

- 9. (10%) Consider the recurrence relation  $a_n = 2a_{n-1} + 3n$ .
  - (a) (2%) Write the associated homogeneous recurrence relation.
  - (b) (2%) Find the general solution to the associated homogeneous recurrence relation.
  - (c) (2%) Find a particular solution to the given recurrence relation.
  - (d) (2%) Write the general solution to the given recurrence relation.
  - (e) (2%) Find the particular solution to the given recurrence relation when  $a_0 = 1$ .
- 10. (10%)
  - a) (5%) Draw all non-isomorphic trees with 5 vertices.
  - b) (5%) Draw all non-isomorphic rooted trees with 4 vertices.