科 目	工程數學	適 用 系 所	土木工程學系結構組、大 地組	時 間	100 分鐘

※請務必在答案卷作答區內作答。

一.Find the general solution of the following equation

(1) $y' = \dfrac{2y}{x}$ (7%) (2) $x^2 y' + 2xy = x - 1$ (8%)

二.Find the general solution of the following equation (10%)

$x^2 y'' + xy' + 4y = \ln x$ $(x > 0)$

三.Given the following integral equation. Find F(t) (10%)
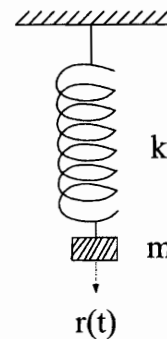
$f(t) = e^{-t} - 2 \int_0^t \cos(t - u) f(u) du$

四.Given the oscillation system, if m=1 c=0 k=25

$r(t) = \begin{cases} 1 \dots 0 \le t < 1 \\ 0 \dots\dots t > 1 \end{cases}$ $y(0) = \dot{y}(0) = 0$

(1) Find the governing differential equation
of the system. (5%)

(2) Find the oscillation equation y(t). (10%)

k

m

r(t)

五. Find the eigenvalues and eigenvectors of the following matrix (23%)

$$\begin{bmatrix} 30 & 0 & -20 \\ 0 & 20 & 0 \\ -20 & 0 & 0 \end{bmatrix}$$

六. Let $f(x, y, z) = x^2 + y^2 + z^2$, (12%)

(a) Find the **gradient** of function $f(x, y, z)$, **grad** f *or* ∇f

(b) Find the **divergence of grad** f

(c) Find the **curl of grad** f

七. Using Green's theorem, evaluate $\oint_c \left[(x^2 - \cosh y) dx + (y + \sin x) dy \right]$

counterclockwise around the boundary curve *C* of the region *R*, where *C* is the

boundary of the rectangle ： $0 \le x \le \pi,\ 0 \le y \le 1$ (15%)