中原大學 100 學年度 碩士班 入學考試

3 月 19 日 13:30~15:00　　通訊工程碩士學位學程　　誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：工程數學(線性代數、機率)　　（共 2 頁第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者　　■不可使用計算機

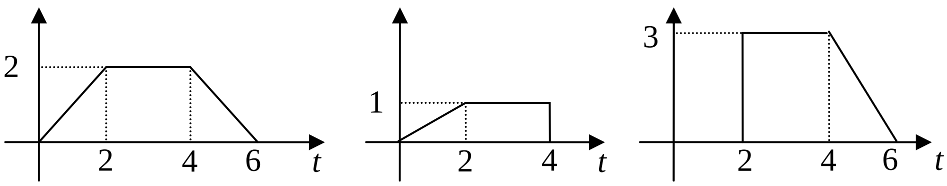
1. $A=\begin{bmatrix}0&1&0&0\\0&0&0&1\\1&0&0&0\\0&0&1&0\end{bmatrix}\begin{bmatrix}1&0&0&0\\-4&1&0&0\\2&4&1&0\\5&2&3&1\end{bmatrix}\begin{bmatrix}5&3&6&5\\0&3&2&1\\0&6&4&5\\0&9&6&3\end{bmatrix}$.

(a) (5%) Find the determinant of A^T.

(b) (5%) Find the rank of *A*.

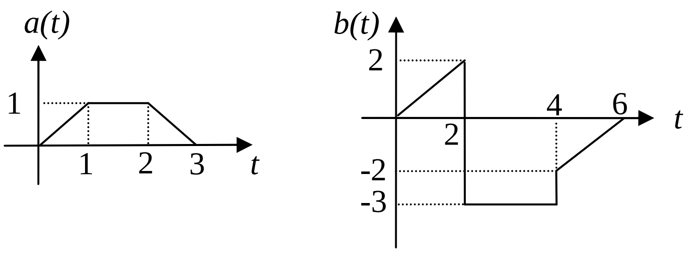
(c) (5%) Is *A* invertible? Explain your answer. Find A^{-1} if it exists.

2. Consider a vector space *V* consisting of all possible linear combinations of the following three functions:



(a) (10%) Use Gram-Schmidt orthogonalization procedure to obtain an orthonormal basis of *V*.

(b) (5%) Which of the two functions shown below is in *V*? Obtain the vector representation of the function with respect to the basis obtained in (a).



3. (a) (10%) Given the following matrix:

$$A=\begin{bmatrix}-5&6&6\\1&-4&-2\\-3&6&4\end{bmatrix}$$

please find constants a_0, a_1, a_2, a_3 with $(a_0, a_1, a_2, a_3) \neq (0,0,0,0)$ satisfying

$$a_0 I + a_1 A^1 + a_2 A^2 + a_3 A^3 = 0$$

where *I* denotes the identity matrix and 0 stands for the zero matrix.

(b) (10%) Given any $n \times n$ matrix B of full rank, please show that we can always find constants $b_0, b_1, \ldots, b_n$ with $(b_0, b_1, \ldots, b_n) \neq (0, 0, \ldots, 0)$ satisfying

$$b_0 I + b_1 B^1 + \cdots + b_n B^n = 0.$$

4. (10%) Consider two joint Gaussian random variables X and Y with the following probability density function $f_{XY}(x, y) = \frac{1}{\sqrt{2}\pi} \exp\left\{-\left(x^2 - \sqrt{2}xy + y^2\right)\right\}$. What is the probability density function of max(X,Y)?

5. (10%) Suppose that the number of earthquakes occurred in a region is modeled as a random variable with Poisson distribution. The mean time between two earthquakes is 6 months. What is the probability that there will be no earthquake within 3 years?

6. (10%) Consider two random variables C and D with positive variances. Please show that

$$E\left[\left(C - E\left[C \mid D\right]\right)\left(D^2 + 2D + 5\right)\right] = 0$$

where E[ ] denotes the operation taking expectation.

7. Let P be a Gaussian random variable with zero mean and unit variance and Q be a discrete random variable with Pr{Q=33}=Pr{Q=88}=0.5 which is independent of P. Consider another random variable R defined by

$$R = \begin{cases} 7P, & \text{if } Q = 33 \\ -7P, & \text{if } Q = 88. \end{cases}$$

(a) (10%) Is R also Gaussian distributed with zero mean? Compute the covariance between P and R.

(b) (10%) Is R independent of P? Does your answer contradict with the fact that uncorrelated joint Gaussian random variables are independent?