中原大學 100 學年度 碩士班 入學考試

3 月 19 日 13:30~15:00　　　企業管理學系乙組　　　誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目： 微積分 　　　　　　　　　　　　　　　　　　（共 1 頁第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者　　　■不可使用計算機

1. 【15%】Find the following limits

    (1). $\lim\limits_{x \to \infty}\left(\sqrt{x^2 + x} - x\right)$,
    (2). $\lim\limits_{x \to 0} \dfrac{x - \sin x}{1 - \cos x}$,
    (3). $\lim\limits_{n \to \infty} \sum\limits_{k=1}^{n} \dfrac{n}{k^2 + n^2}$.

2. 【15%】Evaluate the following integrals if possible.

    (1). $\displaystyle\int_0^3 \dfrac{1}{x-2}\,dx$,
    (2). $\displaystyle\int_0^4 x\sqrt{x^2 + 9}\,dx$,

    (3). $\displaystyle\iint_D e^{\left(x^2 + y^2\right)}\,dxdy$, where $D = \left\{(x, y)\,\middle|\,x^2 + y^2 \leq 1\right\}$.

3. 【10%】Test the following series for convergence or divergence.

    (1). $\displaystyle\sum\limits_{n=2}^{\infty} \dfrac{1}{[\ln(n)]^2}$,
    (2). $\displaystyle\sum\limits_{n=1}^{\infty} \dfrac{1}{n(n+1)}$.

4. 【10%】Use the differentials to estimate $\sqrt{3.99}$.

5. 【20%】Find the extrema of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $z^2 = xy + 1$.

6. 【20%】A liquid from of penicillin manufactured by a pharmaceutical firm is sold in bulk at a price of \$200 per unit. If total production cost for x units is

$$C(x) = 500{,}000 + 80x + 0.003x^2$$

and if the production capacity of the firm is at most 30,000 units in a special time. How many units of penicillin must be manufactured and solid in that time to maximize the profit?

7. 【10%】Let Y_1 and Y_2 have joint density given by

$$f(y_1, y_2) = \begin{cases} 2y_1, & 0 \leq y_1 \leq 1, \quad 0 \leq y_2 \leq 1 \\ 0, & \textit{elsewhere.} \end{cases}$$

Find the expected value of Y_1.