

中原大學 100 學年度碩士班入學考試

3 月 19 日 15:30-17:00 物理學系, 物理學系(在職)

誠實是我們珍視的美德,
我們喜愛「拒絕作弊, 堅守正直」的你!

科目: 應用數學

(共 1 頁第 1 頁)

可使用計算機, 惟僅限不具可程式及多重記憶者 不可使用計算機

1. (20 分) Calculate the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 3 & -2 \\ -2 & 0 \end{pmatrix}$.

2. (a) (10 分) Evaluate the integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ by using the contour integration.

(b) (10 分) Evaluate the integral $\int_0^{\infty} \frac{\cos x dx}{1+x^2}$ by using the contour integration.

3. (20 分) $|a\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $|b\rangle = \begin{pmatrix} -i \\ 1 \end{pmatrix}$ are two vectors in two-dimensional complex vector-space.

Please use "Gram-Schmidt method" to find an orthonormal set of the basis.

4. (a) (10 分) Solve the differential equation $y'' - 6y' + 9y = 0$, where $y' \equiv \frac{dy}{dx}$.

(b) (10 分) Solve the differential equation $y'' + y' - 2y = 4 \sin 2x$, where $y' \equiv \frac{dy}{dx}$.

5. Expand the periodic function $f(x)$ in a sine-cosine Fourier series as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{m=1}^{\infty} b_m \sin mx, \text{ with } f(0)=1.$$

(a) (5 分) find a_0 ,

(b) (5 分) find a_n , ($n \geq 1$)

(c) (10 分) find b_m , ($m \geq 1$)

