中原大學 100 學年度 碩士班 入學考試

3 月 19 日 13:30~15:00　物理學系 、物理學系(在職)　誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：量子物理　（共 2 頁第 1 頁）

□可使用計算機，惟僅限不具可程式及多重記憶者　■不可使用計算機

A. 單選題（20 分，每題 5 分，答案必需依序填於答案卷）

1. The energy density of blackbody radiation is $u(\nu,T)=\frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1}$. At low frequency, this formula can be reduced to ________.

   (A) $u(\nu,T)=-\frac{8\pi h\nu^3}{c^3}$　(B) $u(\nu,T)=\frac{8\pi\nu^2}{c^3}kT$

   (C) $u(\nu,T)=\frac{8\pi h\nu^3}{c^3}e^{-h\nu/kT}$　(D) $u(\nu,T)=\frac{8\pi h^2\nu^4}{c^3 kT}$

2. Consider a nonrelativistic electron whose energy is E. What is its de Broglie wavelength?

   (A) $\frac{h}{E}$　(B) $\frac{E}{h}$　(C) $\frac{h}{\sqrt{2m_eE}}$　(D) $\frac{\sqrt{2m_eE}}{h}$

3. Consider the wavelength change $\lambda'-\lambda=\frac{h}{m_ec}(1-\cos\theta)$ of the Compton scattering. The Compton wavelength of the electron is $\frac{h}{m_ec}=2.426\times10^{-12}$ m. When the scattered photon has the maximum energy loss, what is the wavelength change?

   (A) 0　(B) 1.213×10^{-12} m　(C) 2.426×10^{-12} m　(D) 4.852×10^{-12} m

4. The Hermite polynomials show up in the eigenfunctions of the harmonic oscillator. The polynomials can be expressed as $H_n(y)=(-1)^n e^{y^2}\frac{d^n}{dy^n}e^{-y^2}$. What is $H_3(y)$?

   (A) $8y^3-12y$　(B) $8y^3-4y$　(C) $-8y^3+12y$　(D) $-8y^3+4y$

B. 計算題（80 分，需寫出計算過程）

1. (20 points) The state of a particle is described by the following wave function:

$$\psi(x)=A \quad \text{for} \quad -a<x<2a$$
$$=0 \quad \text{for} \quad x<-a \quad \text{and} \quad x>2a$$

   (a) A is real. Find A by using the normalization condition.
   (b) What is the probability of finding the particle between $x = 0$ and $x = a$?
   (c) Calculate $\langle x\rangle$ and $\langle x^2\rangle$ for this state.
   (d) Calculate the momentum probability density.

2. (20 points) Consider a simple harmonic oscillator system with Hamiltonian $H=\dfrac{p^2}{2m}+\dfrac{m\omega^2x^2}{2}$.

Define the operators A and A^+ as: $A=\sqrt{\dfrac{m\omega}{2\hbar}}x+i\dfrac{p}{\sqrt{2m\omega\hbar}}$ and $A^+=\sqrt{\dfrac{m\omega}{2\hbar}}x-i\dfrac{p}{\sqrt{2m\omega\hbar}}$.

(a) Find the commutation relations $[H,A]$, $[H,A^+]$, and $[A,A^+]$.

(b) When a perturbation λH_1 is introduced, $H\to H+\lambda H_1$. We can expect that the energy eigenvalues $E_n\to E_n+\lambda E_n^{(1)}+\lambda^2E_n^{(2)}...$ Suppose the perturbation is $\lambda H_1=\lambda x$. Calculate the first-order energy shift $\lambda E_n^{(1)}$ and the second-order energy shift $\lambda^2E_n^{(2)}$.

3. (20 points) (a) Use the Schrödinger equation to calculate the eigenfunctions and the energy eigenvalues for an electron in a three-dimensional cubical box with side L.

(b) Consider the Pauli exclusion principle. What is the lowest energy of a set of 25 electrons in a cubical box with side L?

4. (10 points) Consider the addition of spins for a two-electron system. We can denote the spinor of the first electron by $\chi_\pm^{(1)}$, and similarly for the spinor $\chi_\pm^{(2)}$ of the second electron. Use the combinations of $\chi_\pm^{(1)}$ and $\chi_\pm^{(2)}$ to represent the triplet states and the singlet state.

5. (10 points) An electron in a hydrogen atom is in a state described by the wave function

$$\psi(\vec{r})=A\left(\psi_{100}(\vec{r})-2\psi_{200}(\vec{r})+3\psi_{211}(\vec{r})-\sqrt{2}\psi_{210}(\vec{r})\right)$$

(a) Find A by using the normalization condition.

(b) What are the expectation values of L_z and $\vec{L}^2$?