國立東華大學招生考試試題

招生學年度	100	招生類別	碩士班
系所班別	運籌管理研究所碩士班(乙組)		
科目	作業研究		
注意事項	本考科可使用掌上型計算機		

#1. The demands in four markets M_1, M_2, M_3, and M_4 are 5, 9, 4, and 7 units, respectively. They are satisfied by the inventories of three warehouses W_1, W_2, and W_3, with inventory levels of 10, 8, and 7 units, respectively. The cost to ship one unit from a warehouse to a market is given by the table below.

	M_1	M_2	M_3	M_4
W_1	4	4	4	3
W_2	8	6	3	2
W_3	6	5	5	7

(a). (14 points) Formulate an optimization problem to minimize the total shipping cost. It is *not* necessary to solve the problem.

(b). Without knowing what the answer in (a) is, explain the changes in the formulation for the following cases. Each case holds by itself without any effect on the other cases.

(i). (2 points) It is impossible to ship from W_3 to M_2.

(ii). (2 points) Market M_4 must be satisfied by goods from all three warehouses.

(iii). (2 points) Market M_4 must be satisfied by goods from at least two warehouses.

#2. The distances (in miles) between cities A to F are shown below.

from \ to	A	B	C	D	E	F
A	—	112	300	270	157	195
B	112	—	210	170	135	180
C	300	210	—	105	206	306
D	270	170	105	—	140	245
E	157	135	206	140	—	85
F	195	180	306	245	85	—

(a). (10 points) Highways are constructed to connect the six cities A to F such that the cities are connected with minimal total length of highways. What is the appropriate algorithm for this purpose? Briefly describe its procedure. Show how the highways should be

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constructed.

(b). (3 points) For some reasons City B and City E cannot be *directly* linked by a highway. Explain the solution procedure for this case. Give the optimal way to construct the highways.

(c). (2 points) Instead of (b), suppose that City D can only be directly connected to one other city. Explain the solution procedure for this case. Give the optimal way to construct the highways.

#3. (15 points) Use an optimization technique to solve the following mathematical programming problem:

$$\begin{aligned} \min\ & x_1^2 + 2x_2^2 + 3x_3^2, \\ s.t.\ & x_1 + x_2 + x_3 = 6, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

#4. Linear program **LP** is given by:

$$\begin{aligned} \max\ & 3x + 4y, \\ s.t.\ & -x + y \leq 4, \\ & y \leq 6, \\ & x + y \leq 12, \\ & x, y \geq 0. \end{aligned}$$

(a). (10 points) Solve **LP** by the graphical method. Show as much detail as possible in your graph.

(b). (4 points) Find the co-ordinates of all the corner (i.e., extreme) points of the feasible region.

(c). Let s_1, s_2, and s_3 be the slack variables of the three inequality constraints of **LP**, i.e., $s_i \geq 0$ and adding s_i to the ith inequality constraint turns it into an equality constraint, $i = 1, 2, 3$.

(i). (3 points) Find the corner point with x, y, and s_3 as the basic variables.

(ii). (3 points) Is the basic solution with x, y, and s_2 as the basic variables feasible for **LP**?

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#5. (15 points) Re-formulate the following integer program **IP** as a dynamic program and solve it accordingly. Define your terms and notations clearly.

$$\begin{aligned} &\max\ 3x_1 + 5x_2 + 3x_3, \\ &s.t.\ \ 5x_1 + 4x_2 + 3x_3 \le 9, \\ &\qquad x_1, x_2, x_3 \in \{0, 1, 2, \ldots\}. \end{aligned}$$

#6. Consider **IP** in Question #5.

(a). (9 points) Let **LP1** be the linear programming relaxation of **IP**, i.e., the variables x_1, x_2, x_3 in **IP** are relaxed to non-negative real numbers. Solve **LP1** by the simplex method. Show your intermediate work. The optimal solution of **LP1** should *not* be that of **IP** as the optimal basic variable of **LP1** is not an integer.

(b). Suppose that x_i is the optimal basic variable of **LP1**. By construction, x_i is not an integer. Let l and u be the two integers closest to x_i, i.e., $l < x_i < u$, and l is integer part of x_i and u is the integer by rounding up x_i.

(i). (3 points) Add the constraint $x_i \ge u$ to turn **LP1** into **LP2**. Solve **LP2** either by the simplex method or the dual simplex method. You can continue from the optimal simplex tableau of **LP1**, or re-solve **LP2** from scratch.

(ii). (3 points) Instead of (i), add the constraint $x_i \le l$ to turn **LP1** into **LP3**. Solve **LP3** either by the simplex method or the dual simplex method. You can continue from the optimal simplex tableau of **LP1**, or re-solve **LP3** from scratch. Do you get the optimal solution of **IP** from that of **LP3**?

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