

國立交通大學 101 學年度碩士班考試入學試題

科目：工程數學(3091)(3101)

考試日期：101年2月17日 第 1 節

系所班別：土木工程學系

組別：土木系丙組一般生

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【可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. Solve ordinary differential equation (do not use Laplace transform):

$$x'' + 4x' = 4, \quad x(0) = x'(0) = 0. \quad (10\%)$$

2. Solve ordinary differential equation: $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \ln x^3$ (10%)

3. Use Laplace transform to solve the given system equation, subject to the given conditions: (20%)

$$\begin{cases} x'' - 4x' + 8y' + 4y = 4 \\ 2y' - 2x' + y = 0 \end{cases} \quad x(0) = x'(0) = y(0) = 1$$

4. Solve the boundary value problem: (10%)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad \begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq \pi/2 \end{cases}$$

with boundary conditions

$$u(0, y) = 0, u'(x, 0) = 0, u(x, \pi/2) = 0, u(1, y) = g(y), \text{ where } g \text{ is an analytic}$$

function of y . (leaving Fourier coefficient in integral form)

5. A satellite moves with constant speed along a meridian earth and keeps at a height R from the center of the Earth. The angular speed of the Earth rotation is ω and the angular speed of the satellite is σ . Find the velocity (7%) and acceleration of the satellite. (8%)

6. For the given vector fields $\mathbf{v}_1 = [ye^x \quad e^x \quad 2z]$ and $\mathbf{v}_2 = [e^x \quad ye^x \quad e^z]$,

(1) Which vector field can be represented as the gradient of a potential f ? (8%) (Hint: Check if $\text{curl } \mathbf{v} = \mathbf{0}$?)

(2) Find f for the vector field in (a)? (7%)

7. Show that $\phi = C \cosh x \sin y$ is a permissible potential function where C is a constant (5%) and its corresponding stream function that is a conjugate harmonic function corresponding to the potential function. (5%)

8. Heat flows in the direction of maximum decrease of temperature $T = \sin x \cosh y$. Find the direction of the heat flow at a given point $(\pi/4, \ln 5)$. (10%)