

國立臺北大學 111 學年度碩士班一般入學考試試題

系(所)組別：資訊工程學系

科目：線性代數與離散數學

第1頁 共2頁

可 不可使用計算機

1. (10%) Use Gauss-Jordan reduction to solve the following system:

$$\begin{cases} x_1 - 2x_2 - x_3 + 2x_4 = 0 \\ 2x_1 - x_2 + 3x_3 + x_4 = 9 \\ -x_1 + x_2 - x_3 - x_4 = -4 \end{cases}$$

2. Let

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & -1 & 2 \\ 2 & -1 & 3 & 1 \\ -1 & 1 & -1 & -1 \end{bmatrix}.$$

- (a)(10%) Find a basis for the row space of \mathbf{A} and a basis for the null space of \mathbf{A} .

- (b) (5%) Verify the rank-nullity theorem for \mathbf{A} .

3. (10%) Let \mathbf{S} be the two-dimensional subspace of \mathbf{R}^3 spanned by $\mathbf{x}_1 = [1, 0, 2]^T$ and $\mathbf{x}_2 = [0, 1, -4]^T$. Find a basis for \mathbf{S}^\perp .

4. (15%) Let \mathbf{B} be an $m \times n$ matrix, and the dimension of the row space of \mathbf{B} is n . Show that the matrix $\mathbf{B}^T \mathbf{B}$ is symmetric positive definite.

5. (10%) Determine whether the following argument is valid or not valid with a proof.

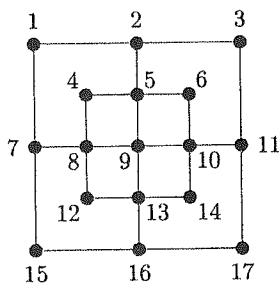
Rainy days make gardens grow.

Gardens don't grow if it is not hot.

It always rains on a day that is not hot.

Therefore, if it is not hot, then it is hot.

6. (10%) A pair of dice, each with the numbers 1, 3, 5, 7, 9, 11, on its six sides are rolled. What is the expected value of the sum of the numbers showing on this pair of dice? What is the expected value of the product of the numbers showing on this pair of dice?
7. (20%) Consider the following graph. Is it a planar graph? Does it have a Euler circuit? Does it have a Euler path? Does it have a Hamilton circuit? Does it have a Hamilton path? Answer these questions and prove your answers.



試題隨卷繳交

接背面

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第2頁 共2頁

可 不可使用計算機

8. (10%) Consider the following recursive function XXX where the global variable *count* is initialized to 0 and input n is a positive integer. What is the final value of *count* after XXX(5) is executed? Furthermore, in general, what is the final value of *count* as a function of n ?

```
XXX( n )
{
  if( n = 1 ) or ( n = 2 ) then
    count ← count + 1
  else
    {
      XXX( n-2 )
      XXX( n-1 )
      XXX( n-2 )
      count ← count + 1
    }
}
```

試題隨卷繳交