國立臺北大學 111 學年度碩士班一般入學考試試題

系（所）組別：統計學系
科　　　目：數理統計

□可 ☑不可使用計算機

1. (15%) Let X_i be an exponential random variable with the probability density function

$$f(x;\lambda_i) = \lambda_i exp(-\lambda_i x), x \geq 0;\ \lambda_i > 0$$

for $i = 1, \ldots, n$. Assume $X_i, i = 1, \ldots, n$, are independent. The detailed derivation has to be provided for all the following problem. No point is given when only answers are given.

(a) (10%) Find $P[X_1 < X_2 < X_3]$ and $P[X_1 < X_2 | \max(X_1, X_2, X_3) = X_3]$.

(b) (5%) Assume $n = 3$, let $X_{(i)}, i = 1,2,3$, denote the order statistics of $X_i, i = 1,2,3$, where $X_{(1)} \leq X_{(2)} \leq X_{(3)}$. Find the marginal distribution of $X_{(3)}$.

2. (35%) Let $X_i, i = 1, \ldots., n$, be an exponential random sample with the probability density function

$$f(x;\lambda) = \lambda exp(-\lambda x), x \geq 0;\ \lambda > 0.$$

Let the distribution function of $f(x;\lambda)$ be denoted as $F(x;\lambda)$. Also, let $X_{(i)}, i = 1,2,\ldots,n$, denote the order statistics of $X_i, i = 1,2,\ldots,n$, where $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$. The detailed derivation has to be provided for all the following problem. No point is given when only answers are given.

(a) (10%) Suppose the sample size N follows a Poisson distribution with parameter μ, where μ>0. Given $N = n > 0$, let $S_n = X_1 + \cdots + X_n$ and $S_0 = 0$. Find the marginal distribution of S_n. [Hint: $P[S_n = 0 | N = 0] = P[N = 0]$.]

(b) (10%) Continued (a), find $E[S_n]$.

(c) (5%) Show that $X_{(1)}$ converges in probability to 0.

(d) (10%) Determine the limiting distribution of $Y_n = X_{(n)} - \lambda^{-1}\log n$.

3. (50%) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a distribution that has pdf

$$f(x) = \theta(1-x)^{\theta-1}, \qquad 0 < x < 1, \theta > 0.$$

(a) (10%) Find the maximum likelihood estimator of $1/\theta$.

(b) (10%) Is the MLE from (a) a complete sufficient statistic?

(c) (10%) Is the MLE from (a) the UMVUE of $1/\theta$? If yes, show it. If not, find the UMVUE of $1/\theta$. Hint: Consider $Y_i = \ln(1 - X_i)$.

(d) (10%) Consider testing

$$H_0: \theta = 1 \text{ against } H_1: \theta \neq 1.$$

(i) (5%) Use Wald test to find the rejection region.

(ii) (5%) Find the likelihood ratio test statistic and write down the rejection criterion.

(e) (10%) True or false. If false, correct the statement.

(i) (5%) The p-value of a test is the type I error.

(ii) (5%) The p-value of a test has a Uniform(0,1) distribution.

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