國立臺北大學 111 學年度碩士班一般入學考試試題

系（所）組別：統計學系
科　　　　目：基礎數學

□可　☑不可使用計算機

一、 Calculus

1. (10 points) Let $f(x)$ be defined in a neighborhood of the origin. Show that if $f'(0)$ exists, then
$$\lim_{h\to 0}\frac{f(h)-f(-h)}{2h}=f'(0).$$
Give a counter example to show that the converse is not true in general, that is, if the above limit exists, then it is not necessary that $f'(0)$ exists.

2. (10 points) Find an equation for the line tangent to the graph of $y=x(\ln x)^2+\dfrac{x}{\ln x}$ at the point $(e,2e)$.

3. (10 points) Let n, r be positive integer. Evaluate
$$\lim_{r\to\infty}\left(\frac{1}{r+1}+\frac{1}{r+2}+\cdots+\frac{1}{r+nr}\right).$$

4. (10 points) Find the continuous function f and constant c such that
$$\int_c^x tf(t)dt=\cos x-\frac{1}{2},\ \forall x\in R.$$

5. (10 points) Sketch the region of integration for the given integral $\int_0^1\int_{x^2}^1 xe^{y^2}dydx$ and evaluate it.

二、 Linear Algebra

1. (20 points) A square matrix is called skew-symmetric if $B^\top=-B$. For any square matrix A, prove that
   a. $A-A^\top$ is skew-symmetric.
   b. the diagonal elements of $A-A^\top$ must be zero.
   c. $\boldsymbol{x}^\top(A-A^\top)\boldsymbol{x}=0$ for all $\boldsymbol{x}\in\Re^n$.
   d. $I_n+A-A^\top$ is invertible.

2. (10 points) Let $A,B\in\Re^{n\times n}$. Prove that
   a. $rank(AB)\le rank(B)$
   b. $rank(AB)\le rank(A)$

3. (10 points) Find an orthogonal basis for $\Re^3$ that contains the vector $\begin{bmatrix}1\\2\\3\end{bmatrix}$.

4. (10 points) Let $\lambda_1,\lambda_2,\ldots,\lambda_n$ be a complete set of eigenvalues of the matrix $A\in\Re^{n\times n}$. Prove that
   a. $det(A)=\lambda_1\lambda_2\cdots\lambda_n$
   b. $tr(A)=\lambda_1+\lambda_2+\cdots+\lambda_n$

試題隨卷繳交