國立臺北大學111學年度碩士班一般入學考試試題

系（所）組別：經濟學系

科　　目：統計學

☑可 ☐不可使用計算機

I. 選擇題（每小題4%，請在作答時標示大題「I」及其小題號（1）、（2）...（5），不須另外標示1、2、3等）：

1. Peter has T = 43 observations on variables y, x_1 and x_2. He estimates the following equation by OLS method:

$$y = 2.0 + 3.0x_1 - 4.0x_2 \ldots\ldots\ldots\ldots\ldots (1)$$
$$(1.5) \quad (-1.0)$$

where the numbers in parentheses are standard errors for coefficients (except for intercept.). Total sum of squares (SST)=100; $R^2 = 0.8$. What is the true value of the sum of squares errors (SSE) in equation (1)? (4 %) ＿＿(1)＿＿

(A) 40
(B) 80
(C) 60
(D) 20

2. Continue Question 1, If Peter tests the following hypothesis: $H_0: \beta_1 = \beta_2 = 0$, what is the value of F-statistic? (4%) ＿＿(2)＿＿

(A) 80
(B) 16
(C) 100
(D) 160

3. Suppose that the true model of y is $y = \beta_1 x_1 + \beta_2 x_2 + u$, but you estimated an incorrect model by OLS:

$$y = \beta_1 x_1 + v \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The true parameter $\beta_2 < 0$, and $E(x_2|x_1)$ has a linear relationship:

$$E(x_2|x_1) = \delta_0 + \delta_1 x_1.$$

If we have the prior information that OLS estimator $\hat{\beta}_1$ of Equation (2) has positive bias, what is the sign of δ_1? (4 %) ＿＿(3)＿＿

(A) $\delta_1 < 0$
(B) $\delta_1 = 0$
(C) $\delta_1 > 0$
(D) cannot decide

4. Suppose you have data (y_t, x_t), t = 1,2, ...,50, with $\sum_{t=1}^{50} x_t = 0$, $\sum_{t=1}^{50} y_t = 100$, $\sum_{t=1}^{50} x_t^2 = 100$, $\sum_{t=1}^{50} y_t^2 = 300$ and $\sum_{t=1}^{50} x_t y_t = 50$. What is the R^2 of this regression? (4%) ____（4）____

(A) 0.125

(B) 0.25

(C) 0.75

(D) 0.032

5. Suppose that there is an exact relationship between random variable y_{it} and its lag value y_{it-1}:

$$y_{it} = \beta_1 y_{it-1} + v_{it}$$

A researcher want to estimate the coefficient β by ordinary least squares (OLS) method. Unfortunately, due to data collection mistakes, she only has time average data. $\overline{y}_{it} = \frac{1}{T}\sum_{t=1}^{T} y_{it}$. If she wants to estimate the following regression by OLS method:

$$\overline{y}_{it} = \beta \overline{y}_{it-1} + \overline{v}_{it},$$

where $\overline{v}_{it} = \frac{1}{T}\sum_{t=1}^{T} v_{it}$. Which of the following characteristics will the OLS estimator $\hat{\beta}$ have? (4%) ___（5）___

(A) unbiased and consistent

(B) unbiased and inconsistent

(C) biased and consistent

(D) biased and inconsistent

II. 計算問答題（配分如標示，請在作答時標示大題「II」及其小題號（6）、（7）...（9），不須另外標示 1-1、1-2 等）：

1. Consider a simple regression model with no intercept:

$$y_t = \beta x_t + u_t.$$

You have two observations (T=2). Suppose x_1 and x_2 are fixed; $\mathrm{E}(u_1) = \mathrm{E}(u_2) = 0$; $\mathrm{var}(u_1) = 1$, $\mathrm{var}(u_2) = 2$, $\mathrm{cov}(u_1 u_2) = 0$.

1-1 (10%) Now consider the estimator $\tilde{\beta} = (x_1 y_1 + \frac{1}{2} x_2 y_2)/(x_1^2 + \frac{1}{2} x_2^2)$.

Whether this estimator is unbiased.___（6）___ Calculate its variance.___（7）___

1-2 (10%) Please compare $\mathrm{var}(\tilde{\beta})$ with $\mathrm{var}(\hat{\beta})$, the variance of least squares estimator $\hat{\beta}$. Does this result contradict the general result that ordinary least squares is efficient? ___（8）___

2. (10%) A linear stationary time series model relating a variable, y_t, to the rational expectation of the, x_t , is given by

$$y_t = \beta E(x_t|I_{t-1}) + u_t,$$

where $E(x_t|I_{t-1})$is the expected value of x_t given all information known up to time t-1. Note that, at a minimum, $(y_{t-1}, x_{t-1}, y_{t-2}, x_{t-2,\ldots})$ is contained in I_{t-1}. Throughout this problem assume that $E(u_t|I_{t-1}) = 0$. For simplicity, there is no intercept in the model. Let $\hat{\beta}$ be the OLS estimator from the regression y_t on x_t, t = 1,2, ..., T. Whether $\hat{\beta}$ is a consistent estimator of β?____(9)____

III. 計算問答題（配分如標示，請在作答時標示大題「III」及其小題號（10）、（11）...（17），不須另外標示1-1、1-2等）：

1. Consider

$$f(y_1, y_2) = \begin{cases} 6(1-y_2) & ,0 \le y_1 \le y_2 \le 1, \\ 0 & ,otherwise \end{cases}$$

1-1 (7%) Find $P(Y_1 \le 3/4, Y_2 \ge 1/2)$ ____(10)____

1-2 (7%) Find $\mathbb{E}(Y_1|Y_2 = y_2)$ ____(11)____

2. The number Y of defects per yard for a certain fabric is known to have a Poisson distribution with parameter λ. However, λ itself is a random variable with probability density function given by

$$f(\lambda) = \begin{cases} e^{-\lambda} & ,\lambda \ge 0, \\ 0 & ,elsewhere. \end{cases}$$

2-1 (7%) Find the unconditional probability function for Y. ____(12)____

2-2 (7%) Find the expected number of defects per yard. ____(13)____

3. In a paper, rock and scissor game, you know your opponent is focusing on playing one of the three actions (paper, rock or scissor). The focused action is 2 times more likely to appear than any one of the other two actions.

3-1 (7%) At the first round the opponent played rock. Based on that sample, what is the likelihood function regarding the opponent type? ____(14)____

3-2 (8%) Before the first round you formulate a null hypothesis that the opponent is rock focusing type and will reject that null hypothesis if the first round he does not play rock. What is the probability of type I error? ____(15)____ What is the probability of type II error against the hypothesis the opponent is a paper focusing type? ____(16)____

3-3 (7%) Before the first round, with no information your belief of the opponent's being paper focusing type, rock focusing type and scissor focusing type are all 1/3. After the first round of his playing rock, what should be your Bayesian updated belief regarding his type probabilities? ____(17)____

試題隨卷繳交