國立交通大學 101 學年度碩士班考試入學試題

科目：工程數學(3051)　　考試日期:101 年 2 月 17 日　第 1 節

系所班別：土木工程學系　　組別：土木系甲組一般生　　第 1 頁, 共 1 頁

【可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. The surface of an ellipsoid is defined as

$$f(x,y,z)=x^2+2y^2+4z^2+xy+3yz-1=0$$

Please find the tangent plane(s) in parallel with the XZ-plane. (20%)

2. Find the eigenvalues and eigenvectors of $\begin{bmatrix} -3 & 0 & 4 & 2 \\ 0 & 1 & -2 & 4 \\ 2 & 4 & -1 & -2 \\ 0 & 2 & -2 & 3 \end{bmatrix}$ (15%)

3. Given $A=\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$, find $\mathbf{A}^{301}$ using a proper similarity transformation. (15%)

4. Solve the following differential equation by power series. (20%)

$$\frac{d^4y}{dx^4}+\sin x\frac{d^2y}{dx^2}=0$$

Note: $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$

5. Solve the following equation in *t*-domain (i.e., using mode-superposition method).

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}+\begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} \sin\omega t \\ 0 \end{Bmatrix}$$

Assume zero initial conditions. (20%)

6. Prove the following relationship in Laplace transform. (10%)

$L\{f(t-a)u(t-a)\}=e^{-as}F(s)$,

where L denotes the Laplace transform operator, u(t) is the unit step function, and $L\{f(t)\}=F(s)$.