【可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. (20%) Find the curvature and radius of curvature of the plane curve defined by

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

where a and b are positive constants, h and k are constants.

2. (20%) It is known that

$$\boldsymbol{\omega}\times\mathbf{u}=(\frac{d\mathbf{R}}{dt}-\frac{d\mathbf{A}}{dt}\mathbf{A})\mathbf{u} \qquad (1)$$

$$\mathbf{R}=\mathbf{I}+\mathbf{A}+\frac{1}{2}\mathbf{A}^2$$

$$\mathbf{A}=\mathbf{A}(t)=\begin{bmatrix}0 & -\phi_3 & \phi_2\\ \phi_3 & 0 & -\phi_1\\ -\phi_2 & \phi_1 & 0\end{bmatrix},\quad \boldsymbol{\omega}=\boldsymbol{\omega}(t)=\begin{Bmatrix}\omega_1\\ \omega_2\\ \omega_2\end{Bmatrix}=\begin{Bmatrix}\omega_1(t)\\ \omega_2(t)\\ \omega_2(t)\end{Bmatrix}$$

where I is the identity matrix of order 3, $\mathbf{u}$ is a 3×1 column matrix, $\boldsymbol{\omega}\times\mathbf{u}$ is the cross product of $\boldsymbol{\omega}$ and $\mathbf{u}$, $\mathbf{A}^2=\mathbf{AA}$, $\phi_1=\phi_1(t)=\sin 2t$, $\phi_2=\phi_2(t)=\sin 3t$, $\phi_3=\phi_3(t)=\sin 5t$.

Let

$$\boldsymbol{\phi}=\boldsymbol{\phi}(t)=\begin{Bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{Bmatrix}=\begin{Bmatrix}\phi_1(t)\\ \phi_2(t)\\ \phi_3(t)\end{Bmatrix},\quad \dot{\boldsymbol{\phi}}=\frac{d\boldsymbol{\phi}}{dt}.$$

From Eq. (1), the relationship between $\boldsymbol{\omega}$ and $\dot{\boldsymbol{\phi}}$ may be expressed by

$$\boldsymbol{\omega}=\mathbf{T}\dot{\boldsymbol{\phi}} \qquad (2)$$

where $\mathbf{T}$ is a 3×3 matrix.

Please determine $\dot{\mathbf{R}}=\frac{d\mathbf{R}}{dt}$ in Eq. (1) and matrix $\mathbf{T}$ in Eq. (2)

3. (20%) Please derive the inverse of a square matrix A which is expressed as follows. It is noted that the determinant A $\neq 0$. (20%)

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

4. (20%) Solve the initial value problem:

$$y''(t) + 3y'(t) - 4y(t) = 10e^{t} - 5e^{2}\delta(t-2), \quad y(0) = 1, \quad y'(0) = 1.$$

5. (20%) Find a general solution for the third-order ODE:

$$x^2 y'''(x) + 4xy''(x) + 2y'(x) = \ln x.$$