國立中正大學
111學年度碩士班招生考試
試 題

[第1節]

科目名稱	工程數學
系所組別	機械工程學系光機電整合工程

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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1. (20%) Please find the general solution for the ordinary differential equation given by

$$\frac{d^2y}{dx^2} - 9y = 3e^{3x}$$

2. (30%) A particle is moving along a cardioid curve C in a two-dimensional plane, whose position can be described in polar coordinates by

$$r = 2(1 - \cos\theta); \quad \theta = \omega t$$

where $\omega > 0$ is a constant. **Hint:** Please express the particle position using a complex function.

(a) (10%) For each instant time, please find the unit vector tangent to C.

(b) (10%) Determine the velocity both in magnitude and direction as function of time.

(c) (10%) Determine the acceleration both in magnitude and direction as function of time.

3. (35%) Consider a set of simultaneous equations as follows.

$$\begin{aligned} 2x_1 - x_2 - x_3 &= 0 \\ -x_1 + 3x_2 - x_3 - x_4 &= 0 \\ -x_1 - x_2 + 3x_3 - x_4 &= 0 \\ -x_2 - x_3 + 2x_4 &= 0 \end{aligned}$$

(a) (5%) Express the 4 equations in the following matrix form

$$Av = 0 \text{ with } v = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

(b) (10%) Verify that the characteristic polynomial of A is $f(s) = s^4 - 10s^3 + 32s^2 - 32s$.

(c) (5%) Determine eigenvalues of A. (Hint. One eigenvalue of A is 2.)

(d) (5%) Are the 4 equations linearly independent? **Why?**

(e) (5%) Show that the solutions of the 4 equations forms a line.

(f) (5%) Determine the 3-dimensional subspace orthogonal to the solution line found in (e).

4. (15%) Consider a vector field $(u(x,y), v(x,y)) = (e^x \sin y, e^x \cos y)$.

(a) (5%) Determine its divergence.

(b) (10%) Explain the physical meaning of the divergence found in (a).