國立中正大學
111學年度碩士班招生考試
試題
[第2節]

科目名稱	線性代數與微分方程
系所組別	電機工程學系－電磁晶片組 電機工程學系－計算機工程組 電機工程學系－電力與電能處理甲組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

Differential Equations

1. (10%) Find the Frobenius series solutions of the differential equation:

$$2x^2y'' + 3xy' - (x^2+1)y = 0$$

2. (10%) Use the *Laplace transform* to solve the integral equation

$$f(t) = \cos t + \int_0^t e^{-\tau} f(t-\tau)d\tau$$

3. (10%) Find a general solution of

$$\mathbf{y}' = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{y} + \begin{pmatrix} e^{2t} \\ -2t \end{pmatrix}, -\infty < t < \infty$$

4. (10%) Solve the initial value problem

$$y'' + 4\pi^2 y = 2\pi\delta(t-2), \quad y(0) = 0,\ y'(0) = 0.$$

5. (10%) **Matrix Exponent**
Use the Laplace transform to compute $e^{\mathtt{A}t}$ for

$$\mathtt{A} = \begin{pmatrix} 3 & 4 \\ 0 & 3 \end{pmatrix}$$

(Continue)

Linear Algebra

6. Let $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 2 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. Find the results with details.

a. (10 %) The eigenvalues and their corresponding eigenvectors.

b. (10 %) Find a matrix $\boldsymbol{P}$ that diagonalizes $\boldsymbol{A}$.

c. (5 %) Find $\boldsymbol{A}^7$.

7. (25 %) Prove that a singular matrix with size $n \times n$ cannot be reduced as $\boldsymbol{I}_n$ by Gauss-Jordan elimination. (Hint: You can use the skill of contradiction to prove this rule)

If you have no idea about this proof, please show a 3×3 matrix without zero row and column as an example to explain this rule.