國立中山大學111學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學【機電系碩士班乙組、丙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學111學年度碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學【機電系碩士班乙組、丙組】　　題號：438001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. Solve the following ODEs.
   (a) $2x \tan y\, dx + \sec^2 y\, dy = 0$ (5%)
   (b) $x^3y''' + xy' - y = x^2$ (5%)

2. Given an Initial Value Problem of a system of ODEs:
   $y_1' = y_1 + 4y_2 - t^2 + 6t,\ y_1(0) = 2.$
   $y_2' = y_1 + y_2 - t^2 + t - 1,\ y_2(0) = -1.$
   The system can be represented as a vector equation $\mathbf{y}' = \mathbf{Ay} + \mathbf{g}$.
   (a) Find eigenvalues and eigenvectors of **A**. (5%)
   (b) General solution for the homogeneous part. (5%)
   (c) Particular solution for the non-homogeneous part. (5%)
   (d) Determine the stability and the type of critical point for the homogeneous part. (5%)

3. Given an ODE: $(1 - x^2)y'' - 2xy' + 2y = 0$
   (a) Find the series solution of the given ODE. (10%)
   (b) Find the arbitrary coefficients in the series solution. (5%)
   (c) Find the two solutions of the series solution respectively. (5%)

4. Using the Laplace transform and showing the details, find the current $i(t)$ in the circuit in Figure 1, assuming zero initial current and charge on the capacitor and:

   $L = 1$ H, $C = 10^{-2}$F, $v = -9900 \cos t$ V if $\pi < t < 3\pi$ and 0 otherwise. (15%)

   Note: $Li'(t) + \dfrac{q(t)}{C} = v(t)$

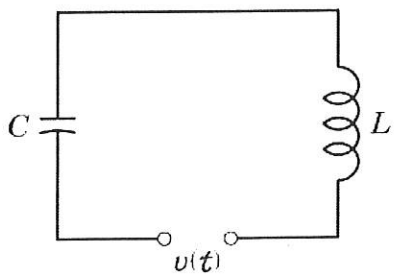

Figure 1.

5. By the principles used in modeling the string it can be shown that small free vertical vibrations of a uniform elastic beam (Figure 2.) are modeled by the fourth-order PDE (equation (1))

$$\frac{\partial^2 u}{\partial t^2} = -c^2 \frac{\partial^4 u}{\partial x^4} \qquad (1)$$

where $c^2 = EI/\rho A$ (E = Young's modulus of elasticity, I = moment of inertia of the cross section with respect to the y-axis in the figure, ρ = density, A = cross-sectional area). Find solutions $u_n = F_n(x)G_n(t)$ of equation (1) corresponding to zero initial velocity and satisfying the boundary conditions for simply supported beam in Figure 3. ($u(0,t) = 0, u(L,t) = 0, u_{xx}(0,t) = 0, u_{xx}(L,t) = 0$). (20%)

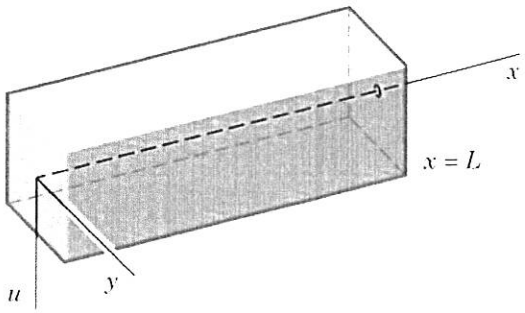

Figure 2.

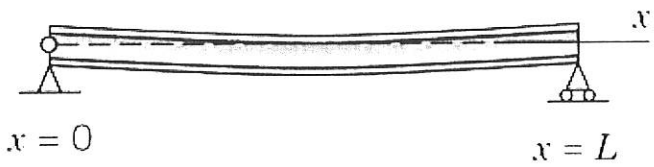

Figure 3.

6. Verify this for **A** and $\mathbf{A} = \mathbf{P}^{-1}\mathbf{AP}$. If **y** is an eigenvector of **P**, show that $\mathbf{x} = \mathbf{Py}$ are eigenvectors of A. Show the details of your work. (15%)

$$\mathbf{A} = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}, \qquad \mathbf{P} = \begin{bmatrix} 0.28 & 0.96 \\ -0.96 & 0.28 \end{bmatrix}.$$