國立中山大學 111 學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學甲【電機系碩士班甲組、己組、庚組、電波聯合選考：電機系碩士班戊組、通訊所碩士班乙組】

—作答注意事項—

考試時間：100 分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，**可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張)。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）

下面 **1-15** 題為單選題，總分 **45** 分。每題答對 **3** 分，答錯扣 **4** 分，未作答者以 **0** 分計。總分低於 **0** 分者以 **0** 分計算。

1. Consider the autonomous differential equation $y' = (2/\pi)y - \sin y$. Which of the following is INCORRECT?
   (A) There are three critical points.
   (B) One of critical point is semi-stable.
   (C) Two of critical points are unstable.
   (D) One of the critical points is 0.

2. If $y = e^{3x}\cos x$ is the solution to $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ky = 0$, what is the value of k?
   (A) 3 (B) -2 (C) 10 (D) 8

3. The differential equation $e^x\frac{dy}{dx} + 3y = x^2y$ is linear and separable.
   (A) True (B) False

4. The improved Euler's method is what type of Runge-Kutta method?
   (A) First order (B) Second order (C) Third order (D) Fourth order

5. Consider $y(x)$ is the solution to the initial-value problem $x^2y'' - 2xy' + 2y = 0$ where $x > 0$, $y(1) = 4$, and $y'(1) = 9$, use Euler's method to compute $y(1.2)$. Given $h = 0.1$, which of the following is correct?
   (A) The general solution is $y = C_1x - C_2x^2$, where $C_1 + C_2 = 6$.
   (B) The general solution is $y = C_1x + C_2x^2$, where $C_1 + C_2 = 6$.
   (C) $y(1.2) = 5.9$.
   (D) $y(1.2) = 6$.

6. Given the three vectors (1, 0, 3, 1), (0, 1, -6, -1) and (0, 2, 1, 0) in R^4, they are linearly dependent.
   (A) True (B) False

7. Provided the system below, the rank is
$$X_1 - X_3 + 2X_4 + X_5 + 6X_6 = -3$$
$$X_2 + 2X_3 + 3X_4 + 2X_5 + 4X_6 = 1$$
$$X_1 - 4X_2 + 3X_3 + X_4 + 2X_6 = 0$$
   (A) 1 (B) 2 (C) 3 (D) 4

8. Which one of the following is correct regarding Fourier series?
   (A) $e^{-|x|}$ is odd function.
   (B) f' must be continuous on the interval $[a, b]$ to ensure that the Fourier series of f on $[a, b]$ converges to f.
   (C) $f(x) = |x|$ is continuous on $[-\pi, \pi]$.
   (D) The Fourier series of $f(x) = x^2 + 1$, where $0 < x < 3$, converges to 0 at $x = 0$.

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9. Expand $f(x)=2x^2-1, -1<x<1$ in a Fourier series and yield $f(x)=A+\sum\frac{B}{n^2\pi^2}C$. Which of the following is correct?
(A) $A=-2/3$　　(B) $B=4$　　(C) $C=(-1)^n\cos n\pi x$　　(D) None of the above

10. If $y_1(x)=x$ is one of the solutions of the following differential equation, what is the other linear independent solution $y_2(x)$?

$$y''-\frac{2x}{1+x^2}y'+\frac{2}{1+x^2}y=0$$

(A) $y_2(x)=2x^2+1$　　(B) $y_2(x)=\frac{x^2-1}{x}$　　(C) $y_2(x)=\frac{1}{x}-1$　　(D) $y_2(x)=x^2-1$

11. Use the Laplace transform to solve the following initial-value problem. If the solution is $y=A+Be^{-t}+Ce^{3t}+De^{4t}$, which of the following is true?

$$y''-4y'=6e^{3t}-3e^{-t}, y(0)=1,\ y'(0)=-1$$

(A) $A+B+C+D=1$.
(B) $B=-2$
(C) $A+B+D=2$
(D) All of the above

12. The Laplace transform of a function f is denoted by $\mathcal{L}\{f\}$. If $\mathcal{L}\{f(t)\}=F(s)$ and $\mathcal{L}\{g(t)\}=G(s)$, then $\mathcal{L}^{-1}\{F(s)G(s)\}=f(t)g(t)$.
(A) True　　(B) False

13. If $\mathcal{L}\{f(t)\}$ represents the Laplace transform of a function $f(t)$. Let $f(t)=\begin{cases}3 & , if\ 0\le t\le 2\\ 5-t, & if\ t>2\end{cases}$, then $\mathcal{L}\{f(t)\}$ is
(A) $\frac{3}{s^2}+\frac{e^{2s}}{s^2}$　　(B) $\frac{3}{s}+\frac{e^{-2s}}{s^2}$　　(C) $\frac{3}{s}-\frac{e^{-2s}}{s^2}$　　(D) $\frac{3}{s^2}-\frac{e^{-2s}}{s^2}$

14. Provided the differential equation $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$, which of the following is true?
(A) first order, linear, non-homogeneous
(B) second order, nonlinear
(C) second order, linear, non-homogeneous
(D) second order, linear, homogeneous

15. The Fourier transform of a function f is denoted by $\Im\{f\}$. Suppose $\Im\{f(t)\}=F(\omega), \Im\{g(t)\}=g(\omega)$, which of the following is INCORRECT?
(A) $\int_{-\infty}^{\infty}f(\tau)g(t-\tau)\,d\tau=\Im^{-1}\{F(\omega)G(\omega)\}$
(B) $\int_{-\infty}^{\infty}f(t-\tau)g(\tau)\,d\tau=\Im^{-1}\{F(\omega)G(\omega)\}$
(C) $\Im\{f(t-\tau)\}=F(\omega)e^{-i\omega\tau}$
(D) None of the above

下面 **16-21** 題為複選題，每題 **5** 分，總分 **30** 分，每題有五個選項，其中至少有一個是正確答案，答錯 **1** 個選項者，得 **3** 分，答錯 **2** 個選項者，得 **1** 分，答錯多於 **2** 個選項或未作答者，該題以零分計算。

16. Let $\mathbf{A}$ and $\mathbf{B}$ be matrices in $\mathbb{R}^{n\times n}$. Which of the following statements are true?
(A) $\det(-\mathbf{A}) = -\det(\mathbf{A})$.
(B) If $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, then $\det(\mathbf{A}) = 1$.
(C) If $\mathbf{A}\mathbf{A}^T = \mathbf{I}$, then $\mathrm{trace}(\mathbf{A}) = n$.
(D) If two rows of $\mathbf{A}$ are equal, then $\det(\mathbf{A}) = 0$.
(E) If $\det(\mathbf{A}) = \det(\mathbf{B})$, then $\mathbf{A}$ and $\mathbf{B}$ have the same rank.

17. Let $\mathbf{A} \in \mathbb{R}^{3\times 3}$ and its eigenvalues are λ_1, λ_1, and λ_2, where λ_1 and λ_2 are distinct eigenvalues. Suppose the dimension of $N(\mathbf{A} - \lambda_1\mathbf{I})$ is 1, where $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$. Which of the following statements are true?
(A) λ_1 must be a real number (not a complex number).
(B) λ_2 must be a real number (not a complex number).
(C) The dimension of $N(\mathbf{A} - \lambda_2\mathbf{I})$ equals 1.
(D) $\mathbf{A}$ is diagonalizable.
(E) $\mathbf{A}$ has two linearly independent eigenvectors corresponding to λ_1.

18. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$. Consider the linear equation $\mathbf{Ax} = \mathbf{b}$ or the homogeneous linear equation $\mathbf{Ax} = \mathbf{0}$. Which of the following statements are true?
(A) If $\mathrm{rank}(\mathbf{A}) = m$, then $\mathbf{Ax} = \mathbf{b}$ has at least one solution for any $\mathbf{b} \in \mathbb{R}^m$.
(B) If $\mathrm{rank}(\mathbf{A}) = m$, then $\mathbf{Ax} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.
(C) If $\mathrm{rank}(\mathbf{A}) = n$, then $\mathbf{Ax} = \mathbf{b}$ has at most one solution for any $\mathbf{b} \in \mathbb{R}^m$.
(D) If $\mathrm{rank}(\mathbf{A}) = n$ and $m > n$, then $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions.
(E) If $\mathrm{rank}(\mathbf{A}) = m$ and $n > m$, then $\mathbf{Ax} = \mathbf{0}$ has infinitely many solutions.

19. Let $\mathbf{A}$ and $\mathbf{B}$ be square matrices. Suppose that $\mathbf{A}$ is similar to $\mathbf{B}$, that is, $\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ for some nonsingular matrix $\mathbf{P}$. Which of the following statements are true?
(A) If $\mathbf{x}$ is an eigenvector of $\mathbf{B}$, then $\mathbf{x}$ is also an eigenvector of $\mathbf{A}$.
(B) If $\mathbf{y}$ is in the column space of $\mathbf{B}$, then $\mathbf{y}$ is also in the column space of $\mathbf{A}$.
(C) $\mathrm{trace}(\mathbf{A}) = \mathrm{trace}(\mathbf{B})$.
(D) $\mathbf{A} - \mathbf{I}$ is similar to $\mathbf{B} - \mathbf{I}$.
(E) $\mathbf{A}^5$ is similar to $\mathbf{B}^5$.

20. Let $\mathbf{A} \in \mathbb{R}^{m\times n}$, $R(\mathbf{A})$ denotes the column space of $\mathbf{A}$, $N(\mathbf{A})$ denotes the null space of $\mathbf{A}$, and $\dim(S)$ denotes the dimension of a subspace S. Which of the following statements are true?
(A) If $\mathbf{y} \in R(\mathbf{A})$, then $\mathbf{y} \in R(\mathbf{A}\mathbf{A}^T)$.
(B) If $\mathbf{x} \in N(\mathbf{A})$, then $\mathbf{x} \in N(\mathbf{A}\mathbf{A}^T)$.
(C) $\mathrm{rank}(\mathbf{A}) + \dim(N(\mathbf{A})) = \mathrm{rank}(\mathbf{A}^T) + \dim(N(\mathbf{A}^T))$.
(D) It is possible for a matrix $\mathbf{A}$ to have $[2, 1, -1]^T$ in $N(\mathbf{A})$ and $[1, -2, 3]^T$ in $R(\mathbf{A}^T)$.
(E) Let $\mathbf{y} \in \mathbb{R}^m$. If $\mathbf{y} = \mathbf{u}_1 + \mathbf{v}_1 = \mathbf{u}_2 + \mathbf{v}_2$, where $\mathbf{u}_1, \mathbf{u}_2 \in R(\mathbf{A})$ and $\mathbf{v}_1, \mathbf{v}_2 \in N(\mathbf{A}^T)$, then $\mathbf{u}_1 = \mathbf{u}_2$ and $\mathbf{v}_1 = \mathbf{v}_2$.

21. Let

$$\mathbf{A} = \begin{bmatrix} 4 & 1 & 3 & 2 \\ 1 & 4 & 3 & 3 \\ -1 & 11 & 6 & 7 \end{bmatrix}.$$

Which of the following vectors are in the column space of $\mathbf{A}$?

(A) $[3,1,2]^T$
(B) $[1,0,-1]^T$
(C) $[0,1,3]^T$
(D) $[2,1,1]^T$
(E) $[4,2,-1]^T$

以下第22題到第23題需要詳明推導計算過程。如推導計算過程錯誤，將酌扣分數或不給分。

22. (10分) 求出以下複平面上之路徑積分值，z 為複數。

$\int_C \frac{z^5}{1-z^3}dz$，其中 C 為沿著 $\{z:|z|=2\}$ 正向旋轉一周之封閉路徑。

23. (15分) 利用餘值（residues）求取以下瑕積分，其中參數$a>0$。

$$\int_0^\infty \frac{\cos ax}{x^2+1}dx$$