# 國立成功大學
# 111學年度碩士班招生考試試題

編　號：102

系　所：土木工程學系

科　目：工程數學

日　期：0219

節　次：第 3 節

備　註：可使用計算機

※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1.　Please solve the following problems: (25%)

$$f = 2x^2 + y^2 + z^2$$
$$\mathbf{u} = xz\mathbf{i} + yz\mathbf{k}$$

Find　(1)　div($f\mathbf{u}$)　(5%)
　　　(2)　$\nabla^2 f$　(5%)
　　　(3)　curl(grad $f$)　(5%)
　　　(4)　div(curl $\mathbf{u}$)　(5%)
　　　(5)　grad($\mathbf{u} \cdot \mathbf{u}$)　(5%)

2.　Please solve the following ODEs: (20%)

(1) Find $\dfrac{dx}{dt} + \dfrac{x}{t} = 2\cos(t)$ for $x(t)$ (10%)

(2) Find $\dfrac{dy}{dx} = 9y^2 x^4$ with $y(2) = 6$ for $y(x)$ (10%)

3.　Let $f(t) = \begin{cases} 0 & -\pi < t < 0 \\ \sin 2t & 0 < t < \pi \end{cases}$ .(20%)

(1) Find the Fourier series of $f(x)$　(15%)

(2) Find Fourier Integral. (5%)

4.　For the given matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find the eigenvalues and eigenvectors of $A$. (15%)

5.　Please solve the following PDEs: (20%)

(1) $\dfrac{\partial u}{\partial x} = 72 \dfrac{\partial u}{\partial t} + 10u$　if　$\dfrac{\partial u}{\partial x}(t = 0) = 6e^{-2x}$. (10%)

(2) $x^2 \dfrac{\partial u}{\partial x} + y^3 \dfrac{\partial u}{\partial y} = 0$　if　$u(0, y) = 5e^{\frac{2}{x}}$. (10%)

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編　號：103

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科　目：工程統計

日　期：0219

節　次：第 3 節

備　註：可使用計算機

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1. Consider a group of five potential blood donors—*a, b, c, d,* and *e*—of whom only *a* and *b* have type O⁺ blood. Five blood samples, one from each individual, will be typed in random order until an O⁺ individual is identified. Let Y denote the number of typings necessary to identify an O⁺ individual. (5 points each)

    (1) What is the probability mass function (pmf) of Y?

    (2) What is the cumulative distribution function (CDF) of Y?

    (3) What is the expected value of Y? Why does the average number make sense in this case?

    (4) What is the variance of Y?

2. Let X denote the vibratory stress (psi) on a wind turbine blade at a particular wind speed in a wind tunnel. The following distribution is proposed as the probability density function (pdf) of X. (5 points each)

$$f(x, \theta) = \begin{cases} \dfrac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} & x > 0 \\ 0 & otherwise \end{cases}$$

    (1) Verify that f(x, θ) is a legitimate pdf?

    (2) Suppose θ=100. What is the probability that X is at most 200? Less than 200? At least 200?

    (3) What is the probability that X is between 100 and 200 (again assuming θ=100)?

    (4) Give an expression for $P(X \le x)$

3. A manufacturer of electric device for construction measurements has a warehouse filled with newly manufactured components. Let μ and σ² denote the mean and variance, respectively, of the population of the lifetimes (in years) of all components in the warehouse. From previous studies, a value of 5.25 is available for σ². (5 points each)

    (1) What sample size should be used if it was desired to estimate to be within 1 year with 99% confidence? (Assume that the lifetimes are normally distributed for this part)

    (2) Suppose that a quality control engineer selected 144 components at random from the warehouse and measured the lifetime of each (in years) and calculated the sample mean to be 12.13. Compute the 95% confidence interval for μ.

    (3) Explain what you mean by saying that you have 95% confidence in this interval.

    (4) Perform a test of hypotheses $H_0 : \mu=12$ versus $H_a : \mu \ne 12$ using α=0.05. State your conclusion.

Values Provided for Your Calculations

| z | 1.64 | 1.96 | 2.58 | $t_{0.025, 48}$ | $t_{0.025, 49}$ | $t_{0.025, 50}$ | $F_{0.05,4,20}$ | $F_{0.05,4,21}$ | $F_{0.05,4,22}$ | $F_{0.05,3,20}$ | $F_{0.05,3,21}$ | $F_{0.05,3,22}$ |
|---|------|------|------|------|------|------|------|------|------|------|------|------|
| Φ(z) | 0.95 | 0.975 | 0.995 | 2.011 | 2.009 | 2.008 | 2.87 | 2.23 | 2.22 | 3.10 | 3.07 | 2.35 |

4. Two job designs, i.e., A and B, are being considered for the production of new computer desks. Manufactures would like to understand whether or not there is a difference in assemble times of these two designs. Two samples are randomly and independently selected for 25 workers to assemble desks using design A, and for the same 25 workers to assemble desks using design B. The assembly times are recorded in minutes, and resulted in the accompanying data. (5 points each)

| Design | Sample Size | Average | Variance |
|--------|-------------|---------|----------|
| Type A | 25 | 6.288 | 0.8481 |
| Type B | 25 | 6.016 | 1.3020 |

(1) State and test the relevant hypotheses using $\alpha=0.05$.

(2) Do the assembly times of the two designs differ based on your answer in (1)? Why?

(3) Make a 95% confidence interval for the difference in the assembly times.

(4) Does there exist a difference in assembly times based on your answer in (3)? Why?

Note:

$$S_p^2 = \frac{m-1}{m+n-2}S_m^2 + \frac{n-1}{m+n-2}S_n^2$$

5. The data on calcium content of wheat is observed. Four different storage times are considered with 6 observations taken for each storage period. The following table lists the test results.　Is there sufficient evidence to conclude that the mean calcium content is the same for the four different storage times?　Use $\alpha=0.05$.

| Storage Period (month) | Mean | Standard Deviation | Frequency |
|------------------------|------|--------------------|-----------|
| 0 | 57.5 | 1.3 | 6 |
| 1 | 58.0 | 1.3 | 6 |
| 2 | 59.6 | 0.9 | 6 |
| 4 | 60.3 | 1.5 | 6 |

(1) Complete the ANOVA table by filling the blanks below. (2 points each)

| Source | SS | DF | MS | F |
|--------|----|----|----|----|
| Treatment | 30 | (b) | (e) | (g) |
| Error | (a) | (c) | (f) | |
| Total | 70 | (d) | | |

(2) Using the ANOVA table to make your conclusion.　Make sure to include in your answer the null and alternative hypotheses. Using $\alpha = 0.05$ (2 points)

(3) What are the assumptions made for the above ANOVA table? (4 points)