國立成功大學

111學年度碩士班招生考試試題

編　號：115

系　所：工程科學系

科　目：工程力學

日　期：0220

節　次：第 1 節

備　註：可使用計算機

※考生請注意：本試題可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

工程力學共五題，請詳細撰寫計算與推導過程，並將最終答案以底線標註。

1. (20%) A three dimensional vector $\mathbf{a}$ can be written as follows:

$$\mathbf{a} = a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3 \quad \text{or} \quad \mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix},$$

where $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are three base vectors of the Cartesian coordinate system. The frame as shown in Figure 1 is subjected to a force $\mathbf{F} = F_1\mathbf{e}_1 + F_2\mathbf{e}_2 + F_3\mathbf{e}_3$ at the vertex D. The position vectors of the vertex D and the feet A, B, C are $\mathbf{x}^D$, $\mathbf{x}^A$, $\mathbf{x}^B$, and $\mathbf{x}^C$, respectively, where

$$\mathbf{x}^A = x_1^A\mathbf{e}_1 + x_2^A\mathbf{e}_2 + x_3^A\mathbf{e}_3,$$
$$\mathbf{x}^B = x_1^B\mathbf{e}_1 + x_2^B\mathbf{e}_2 + x_3^B\mathbf{e}_3,$$
$$\mathbf{x}^C = x_1^C\mathbf{e}_1 + x_2^C\mathbf{e}_2 + x_3^C\mathbf{e}_3,$$
$$\mathbf{x}^D = x_1^D\mathbf{e}_1 + x_2^D\mathbf{e}_2 + x_3^D\mathbf{e}_3.$$

(a) Please determine the moment $\mathbf{M}$ of the force $\mathbf{F}$ about the point A. (10 %)
(b) Please determine the magnitude of the components of the force parallel and perpendicular to member AD. (10 %)

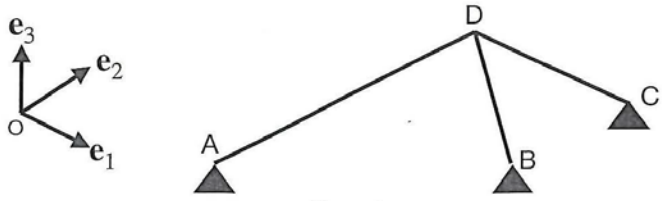

Figure 1

2. (20%) Figure 2 shows a pipe assembly which is subjected two forces $\mathbf{F}^A = x_1^A\mathbf{e}_1 + x_2^A\mathbf{e}_2 + x_3^A\mathbf{e}_3$ and $\mathbf{F}^B = x_1^B\mathbf{e}_1 + x_2^B\mathbf{e}_2 + x_3^B\mathbf{e}_3$ at points A and B, respectively, and the position vectors of the collinear points A, B and C are $\mathbf{x}^A$, $\mathbf{x}^B$, and $\mathbf{x}^C$ where

$$\mathbf{x}^A = x_1^A\mathbf{e}_1 + x_2^A\mathbf{e}_2 + x_3^A\mathbf{e}_3,$$
$$\mathbf{x}^B = x_1^B\mathbf{e}_1 + x_2^B\mathbf{e}_2 + x_3^B\mathbf{e}_3,$$
$$\mathbf{x}^C = x_1^C\mathbf{e}_1 + x_2^C\mathbf{e}_2 + x_3^C\mathbf{e}_3.$$

(a) Please determine the equivalent resultant force $\mathbf{F}_R$ and the resultant moment $\mathbf{M}_{RC}$ about the point C. (10 %)

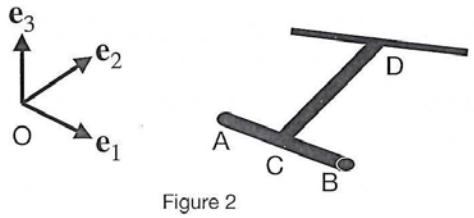

Figure 2

(b) Figure 3 shows two such pipe assemblies, where are subjected forces $\mathbf{F}^A$, $\mathbf{F}^B$, $\mathbf{F}^E$, and $\mathbf{F}^F$ at points A, B, E, and F, respectively, are installed to a pipe PQ. If the bending moment between DH is constant, please determine the forces $\mathbf{F}^A$, $\mathbf{F}^B$, $\mathbf{F}^E$, and $\mathbf{F}^F$ and the positions $\mathbf{x}^A$, $\mathbf{x}^B$, $\mathbf{x}^E$, and $\mathbf{x}^F$. (10 %)

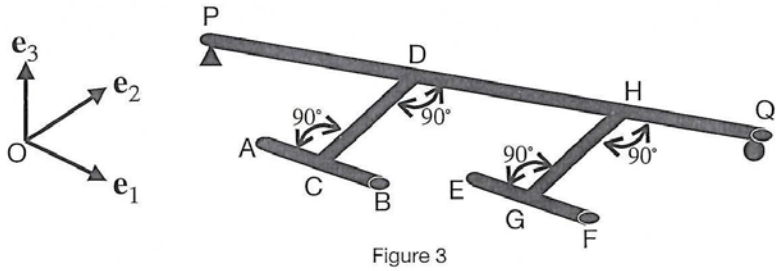

Figure 3

3. (20 %) A motion of a particle with the mass m has the position which is the function of time as follows:

$$\mathbf{x}(t) = r\cos\omega t\,\mathbf{e}_1 + r\sin\omega t\,\mathbf{e}_2 + \alpha t\,\mathbf{e}_3,$$

where t denotes the time and ω, α are two constants. For this motion,

(a) please show the velocity $\mathbf{v}$, the speed v, and the acceleration $\mathbf{a}$; (5 %)

(b) please show the arc length s and derive the curvature κ and base vectors of the natural coordinate system including the tangent vector $\mathbf{t}(s)$, the normal vector $\mathbf{n}(s)$, and the bi-normal vector $\mathbf{b}(s)$; (10 %)

(c) please find the force $\mathbf{F}$ which exerts the motion. (5 %)

4. (20 %) A particle has a velocity $\mathbf{v}' = v_1'\mathbf{e}_1' + v_2'\mathbf{e}_2' + v_3'\mathbf{e}_3'$ measured relative to a rotated Cartesian coordinate system with angular velocity $\boldsymbol{\omega} = \omega_1\mathbf{e}_1 + \omega_2\mathbf{e}_2 + \omega_3\mathbf{e}_3$, where $\mathbf{e}_1', \mathbf{e}_2', \mathbf{e}_3'$ are three Cartesian base vectors of the rotated coordinate system and $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are three base vectors of the fixed coordinate system.

(a) What is the velocity $\mathbf{v}$ of the particle measured relative to a fixed coordinate system? Please give the detail about the derivation. (5 %)

(b) What is the acceleration $\mathbf{a}$ measured relative to a fixed coordinate system? Please give the detail about the derivation. (5 %)

(c) A bullet is fired from a car with an initial velocity $\mathbf{v}' = v_2'\mathbf{e}_2'$ where $\mathbf{e}_2'$ denotes the direction of the car which rotates with $\boldsymbol{\omega} = \omega_3\mathbf{e}_3$. What is the bullet's instantaneous velocity $\mathbf{v}$ and its instantaneous acceleration $\mathbf{a}$ expressed from the fixed coordinate system? Please detail every component of the velocity and the acceleration and please also point out the term "Coriolis acceleration." (10 %)

The position of the car and the fixed coordinate system $\mathbf{e}_i$ are plot in Figures 4 and 5.

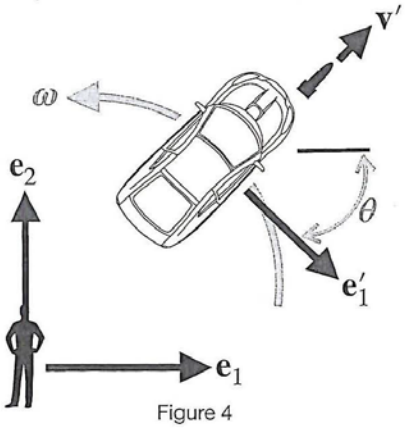

Figure 4

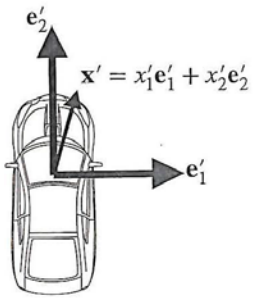

Figure 5

5. (20 %) Consider a damped vibration system with a mass m, a k-spring, and a c-damper and the equilibrium position of the mass is $u = 0$ as shown in Figure 6. The damped vibration system locates on a car with a traveling displacement u_g.

(a) If the car is not traveling, $u_g = 0$, please find the system's equation of motion, (3 %)

(b) and please derive its solution of displacement $u(t)$ for the over-damped case under the initial displacement $u(0) = u_0$ and the initial velocity $\dot{u}(0) = v_0$. (12 %)

(c) If the car is traveling, please find the system's equation of motion. (5 %)

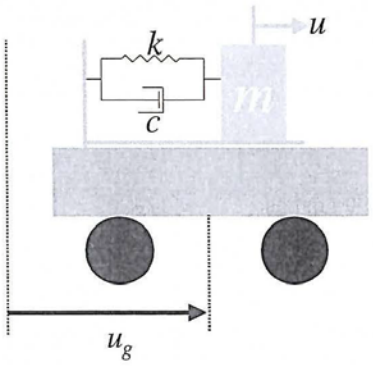

Figure 6