

國立成功大學

111學年度碩士班招生考試試題

編 號： 121

系 所： 工程科學系

科 目： 計算機數學

日 期： 0220

節 次： 第 3 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1、 Let $A = \begin{bmatrix} 1 & -1 \\ 1 & -\frac{3}{2} \end{bmatrix}$

(a) Compute eigenvalues and the corresponding eigenvectors of A. (7%)

(b) Compute $\lim_{n \rightarrow \infty} (A)^{2n+1}$ (8%)

2、 Assume that the function f satisfies the recurrence relation $f(n) = 2f(\sqrt{n}) + \log_2 n$ whenever n is a perfect square greater than 1 and $f(2) = 1$.

(a) Compute $f(30)$ (5%)

(b) Compute a big-O estimate for $f(n)$ (10%)

3、 Solve the recurrence relation $T_n = 3T_{n-1} - 3T_{n-2}$ with boundary condition $T_1 = 6$ and $T_2 = 20$. (10%)

4、 Let $A = \begin{bmatrix} 1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$

(a) Compute a matrix P that diagonalizes A (5%)

(b) Compute $P^{-1}AP$ (5%)

(b) Compute A^5 (5%)

5、 Prove or disapprove that given an integer n , if n^3+5 is odd, then n is odd. (10%)

6、 Select four integers from the first 10 natural numbers randomly. What is the probability that the second smallest of these four chosen numbers is 6? (10%)

7、 How many ways to put 7 different balls into 4 different boxes such that no box is allowed to be empty? (10%)

8、 If R is transitive. Prove or disprove R^3 is transitive or not transitive. (15%)