

國立成功大學

111學年度碩士班招生考試試題

編 號： 111

系 所： 工程科學系

科 目： 線性代數與機率

日 期： 0220

節 次： 第 3 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) A  $4 \times 4$  matrix  $D$  is given by

$$D = \begin{pmatrix} 1 & a & b & c \\ 1 & a^2 & b^2 & c^2 \\ 1 & a^3 & b^3 & c^3 \\ 1 & a^4 & b^4 & c^4 \end{pmatrix}.$$

Please express the determinant  $\det(D)$  in terms of  $a, b$ , and  $c$ .

2. (10%) Find  $\det(D)$  if the eigenvalues of a  $5 \times 5$  matrix  $D$  are 1, 2, 3, 4, and 5.

3. (15%) Let  $A$  be a binary  $6 \times 10$  matrix given by

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & & & & & & \\ 1 & 1 & 0 & 0 & & & & & & \\ 0 & 0 & 1 & 0 & & & & & & \\ 1 & 0 & 1 & 1 & & & & & & \\ 0 & 1 & 0 & 1 & & & & & & \\ 1 & 1 & 0 & 1 & & & & & & \end{pmatrix} I_6$$

where  $I_6$  is the  $6 \times 6$  identity matrix. Please find a  $4 \times 10$  matrix  $B$  such that  $BA^T = 0$ .

4. (10%) Let  $X$  be a lognormal random variable with parameters  $\mu = 0$  and  $\sigma^2 = 1$ , that is,  $\ln X \sim N(0, 1)$ . Find the variance of  $X$ .
5. (15%) Let  $X$  be a geometric random variable with parameter  $p$ . Also let  $k$  and  $n$  be positive integers.
- (5%) Find  $P(X < k + n | X > n)$ .
  - (5%) Find  $P(X < k)$ .
  - (5%) Show that  $X$  is a memoryless random variable.
6. (15%) Let  $X_1, X_2, \dots, X_n$  be independent exponential random variables with parameters  $\lambda_1, \lambda_2, \dots, \lambda_n$ , respectively. Also let  $X = \min(X_1, X_2, \dots, X_n)$ .
- (5%) Find the moment-generating function of  $X_1$ .
  - (5%) Find the cumulative distribution function of  $X$ .
  - (5%) Find the moment-generating function of  $X$ .
7. (10%) Suppose there are 20 students and each student has one pen. Now they all put their pens into a box. Then, all of them randomly pick up a pen from the box. Find the expected number of students to get their own pen back.
8. (15%) Let  $X$  be a binomial random variable with parameters  $(n, p)$ . Find  $E(X^3)$ .