國立成功大學

111學年度碩士班招生考試試題

編　號：42

系　所：光電科學與工程學系

科　目：工程數學

日　期：0220

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Find the eigenfunctions and the equation that defines the eigenvalues for the boundary-value problem.

$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(1) + y'(1) = 0$

2. (10%) Solve

$y'(t) = \cos t + \int_0^t y(\tau)\cos(t-\tau)d\tau, \quad y(0) = 1$

3. (15%) Let

$\mathbf{A} = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ and $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

Solve

$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x}, \quad \text{with } \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

4. (15%) Use separation of variables to find product solutions for

$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad k > 0$

5. (15%) (a) Find the inverse of $\mathbf{A} = \begin{pmatrix} 2 & 2 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & 1 \end{pmatrix}$, and find the diagonal matrix of $\mathbf{B} = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$.

(b) Prove $\nabla \cdot (\nabla \times \boldsymbol{F}) = 0$, if $\boldsymbol{F}$ is a vector field having continuous second partial derivatives.

6. (8%) Evaluate $\oint_c (x^5 + 3y)dx + (2x - 2y^3 e^y)dy$, where C is the circle $(x-1)^2 + (y-5)^2 = 4$.

7. (12%) Evaluate the Cauchy principal value of $\int_{-\infty}^{\infty} \frac{1}{x^4+1} dx$.

8. (15%) (a) Verify $f(s) = \frac{1}{i\pi}[P.V. \int_{-\infty}^{\infty} \frac{f(x)}{x-s} dx]$. (*P.V.* means Cauchy principal value and s is real.)

(b) Suppose $f(\mathrm{x}) = f_1(\mathrm{x}) + \mathrm{i} f_2(\mathrm{x})$, in which $f_1(\mathrm{x})$ and $f_2(\mathrm{x})$ are the real and imaginary parts of $f(\mathrm{x})$, respectively, and $f_2(\mathrm{x})$ is odd function. Find $f_1(\mathrm{s}) = \frac{2}{\pi}[P.V. \int_0^{\infty} \frac{x f_2(x)}{x^2 - s^2} dx]$.