# 國立成功大學
# 111學年度碩士班招生考試試題

編　號：37

系　所：物理學系

科　目：物理數學

日　期：0220

節　次：第 1 節

備　註：不可使用計算機

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# 國立成功大學 111 學年度碩士班招生考試試題

系　　所：物理學系

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考試日期：0220，節次：1



※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. The displacement vector is given by $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. (15%)
   (a) Compute the divergence and the curl of $\vec{r}$. (10%)
   (b) Using the divergence theorem $\iiint_V \nabla \cdot \vec{V} d\tau = \iint_{\partial V} \vec{V} \cdot \hat{n} d\sigma$, calculate $\iint \vec{r} \cdot \hat{n} d\sigma$ over the entire surface of the cone with base $x^2 + y^2 \le 16$, $z = 0$, and vertex at $(0,0,3)$. (5%)

2. Find the solution $y(x)$ of the differential equation $d^2y(x)/dx^2 + 9y(x) = 30\sin(3x)$. (10%)

3. The matrix $M = \frac{1}{2}\begin{pmatrix} -\sqrt{3} & 1 \\ -1 & -\sqrt{3} \end{pmatrix}$ represents an active transformation of vector in the $(x, y)$ plane (axes fixed, vectors rotated and reflected). (20%)
   (a) Find the determinant of this matrix. (5%)
   (b) Calculate $M^T M$ and show that the matrix is orthogonal. (10%)
   (c) Find the rotation angle, or find the line of reflection. (5%)

4. Consider the function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. (25%)
   (a) Find the sine-cosine Fourier series of $f(x)$. (10%)
   (b) Evaluate the result of (a) at $x = 0$, and calculate $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (5%)
   (c) Evaluate the result of (a) at $x = \pi$. (5%)
   (d) Evaluate the result of (a) at $x = \pi/2$, and calculate $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$. (5%)

5. The integral representation of the gamma function is $\Gamma(p) = \int_0^{\infty} x^{p-1} e^{-x} dx$ with $p > 0$. (10%)
   (a) Express $\int_0^1 t^2 \left(\ln \frac{1}{t}\right)^3 dt$ in terms of the gamma function. (5%)
   (b) Evaluate the result of (a). (5%)

6. Consider the function $f(z) = \frac{1}{z^2 + 4z + 5}$. (20%)
   (a) Locate the poles of $f(z)$ and evaluate the corresponding residues. (10%)
   (b) Calculate the integral $\int_{-\infty}^{\infty} \frac{\sin x dx}{x^2 + 4x + 5}$. (10%)