注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。

國立清華大學 111 學年度碩士班考試入學試題

系所班組別：化學工程學系

科目代碼：0901

考試科目：輸送現象及單元操作

一作答注意事項一

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 考試開始後，請於作答前先翻閱整份試題，是否有污損或試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「↓ 由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

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　　*請在【答案卡】作答

Problem 1 (20%)

Multiple choice: Pick only one answer for each question. Each problem is 2 points.

1. During the COVID-19 pandemic, the global shortage of semiconductor chips has drawn a spotlight on the importance of Taiwan's semiconductor manufacturing industry. The Taiwanese semiconductor firms have successfully established their competitive advantages in terms of cost, quality, speed, and flexibility. Spin coating is a procedure widely used in the semiconductor industry to generate uniform thin films on flat substrates such as wafers. The procedure of the spin coating is to deposit a liquid on a disk with radius R which is then spun at angular velocity ω. Centrifugal force causes the fluid to be thrown out radially, leading to the layer getting thinner over time. Assume the liquid is a Newtonian fluid and the viscosity μ and density ρ of liquid are constant. The spin process is at pseudo-steady-state. There is no slip between the liquid and wafer. The liquid is so thin that gravity and pressure gradient can be ignored. **Please derive the governing equation based on the following equations.**

Equation of continuity

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Equation of motion

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r v_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_r v_\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r v_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$

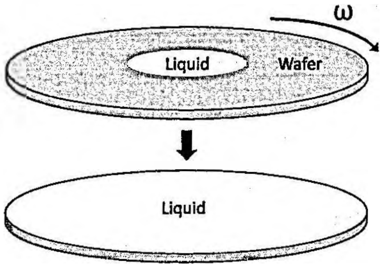
(A) $\rho v_r \frac{\partial v_\theta}{\partial r}=\mu\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r v_\theta}{\partial r}\right)$

(B) $\rho v_r \frac{\partial v_\theta}{\partial r}=\mu\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}$

(C) $\rho\left(v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}\right)=\mu\frac{\partial^2 v_r}{\partial z^2}$

(D) $\rho v_r \frac{\partial v_r}{\partial r}=\mu\left(\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}\right)$

(E) $\rho\left(v_r\frac{\partial v_r}{\partial r}-\frac{v_\theta^2}{r}\right)=\mu\frac{\partial^2 v_r}{\partial z^2}$



2. Assume the coating layer thickness is h. Please choose the **correct** boundary conditions.

(A) $v_r = 0$ at $r = 0$, $\frac{dv_r}{dr} = 0$ at $z = h$　(B) $v_r = 0$ at $z = h$, $\frac{dv_r}{dr} = 0$ at $z = 0$

(C) $v_r = 0$ at $z = 0$, $\frac{dv_r}{dz} = 0$ at $z = h$　(D) $v_r = 0$ at $z = 0$, $\frac{dv_r}{dr} = 0$ at $z = h$

(E) $v_r = 0$ at $z = h$, $\frac{dv_r}{d\theta} = 0$ at $z = 0$

3. Please choose the **correct** fluid velocity in the radial direction of r. The velocity profile in the θ direction is just uniform rotation $v_\theta = \omega r$ for all z.

(A) $v_r = \frac{\rho}{\mu}\omega^2 r\left(-\frac{z^2}{2}+\frac{hz}{2}\right)$, (B) $v_r = \frac{\rho}{\mu}\omega^2 r\left(-\frac{z^2}{2}+hz\right)$, (C) $v_r = \frac{\rho}{\mu}\omega r\left(-\frac{z^2}{2}+\frac{hz}{2}\right)$

(D) $v_r = \frac{\rho}{\mu}\omega r\left(-\frac{z^2}{2}+\frac{h^2 z}{2}\right)$, (E) $v_r = \frac{\rho}{\mu}\omega r\left(-\frac{z^2}{2}+\frac{hz}{4}\right)$

4. With appropriate flow and velocity boundary conditions, and considering a film that is initially uniform, the film thickness as a function of time, $h(t)$, was found to be as follows.

$$h = \frac{h_0}{\sqrt{1+\frac{4h_0^2\rho\omega^2 t}{3\mu}}}$$

where h_0 is the film thickness at time zero and t is the spin time.

Please choose the **WRONG** answer.

(A) The film thickness is the same at the different radius of the wafer.

(B) The film thickness decreases with the spin time.

(C) The film thickness increases with the increase of the fluid viscosity.

(D) The above equation tells us the uniformity of the film can be expected with an irregular initial distribution of the liquid.

(E) The above equation can only be applied to the Newtonian fluid.

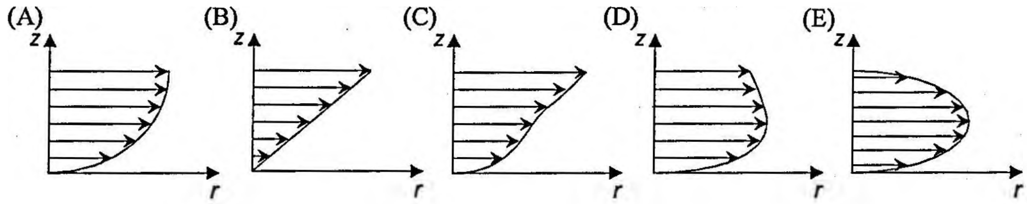
5. Photoresist is a light-sensitive polymer primarily used in the semiconductor process. It can be uniformly coated on the wafer using a spin coater. Assume the photoresist has a viscosity of 5 mPa·s and a density of 1.1 g/mL. The spin speed is set to 4000 rpm and the speed time is 60 s. The initial film thickness is 1 mm. Please estimate the film thickness and **choose the closest value.**

(A) 50 μm, (B) 10 μm, (C) 5 μm, (D) 1 μm, (E) 0.5 μm

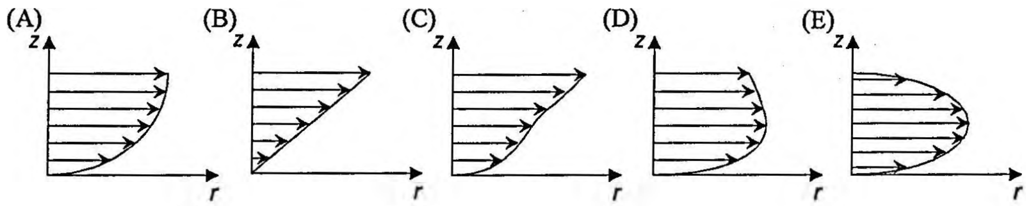
6. Following the above question, please **estimate the percentage** of photoresists flung off from the spinning process. Assume the wafer is 10 inches. The initially applied photoresist film is 4 cm in diameter and 1 mm in height.

(A) 98%, (B) 96%, (C) 80%, (D) 60%, (E) 12%

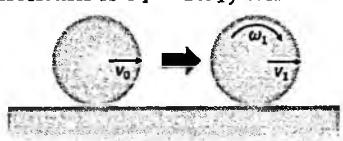
7. A shear-thinning liquid is applied to the wafer and spun using the spin coater. Please choose the **correct** velocity profile during the initial coating process.



8. A high viscosity liquid is applied on the wafer and spun using the spin coater. Please choose the **correct** velocity profile during the initial coating process.



9. Small particles attached to the wafer surface can cause the wafer defect during the spin coating, resulting in a dramatic loss of chip production. To identify the trajectory of the particle, we can simply assume a single spherical particle moving on a surface. The particle has an initial velocity v_0 parallel to the surface with rotational velocity. As soon as the particle moves, its translational momentum is partly converted into angular momentum. Assume no rolling resistance. Please use momentum balance to obtain the velocity v_1 after rolling. [Hint] The angular momentum is $J_l = I\omega_l$, where I is the momentum of inertial of a sphere and $I = \frac{2}{5}mr^2$.



(A) 3/4 v_0, (B) 3/5 v_0, (C) 5/7 v_0, (D) 2/3 v_0, (E) 2/5 v_0

10. A particle submerged in a liquid is spun out from a wafer using a spin coater. Please use centrifugal force and drag force balance to estimate the particle velocity at the edge of the wafer with radius R. Assume the particle diameter is d_p and the density is ρ_p. The liquid has viscosity μ.

(A) $\frac{128\mu R}{d_p^2 \rho_p}$, (B) $\frac{64\mu R}{d_p^2 \rho_p}$, (C) $\frac{36\mu R}{d_p^2 \rho_p}$, (D) $\frac{18\mu R}{d_p^2 \rho_p}$, (E) $\frac{9\mu R}{d_p^2 \rho_p}$

Problem 2 (20%)

Multiple choice: Pick only one answer for each question. Each problem is 2 points.

11. There exist mathematical and physical similarities between momentum and energy transfers. What is the physical quantity in energy transfer that is corresponding to kinematic viscosity in momentum transfer?
(A) temperature gradient
(B) specific heat capacity
(C) thermal diffusivity
(D) thermal conductivity

12. What is the SI unit of heat flux?
(A) J/s, (B) W/m^2, (C) J/m^2, (D) W/s.

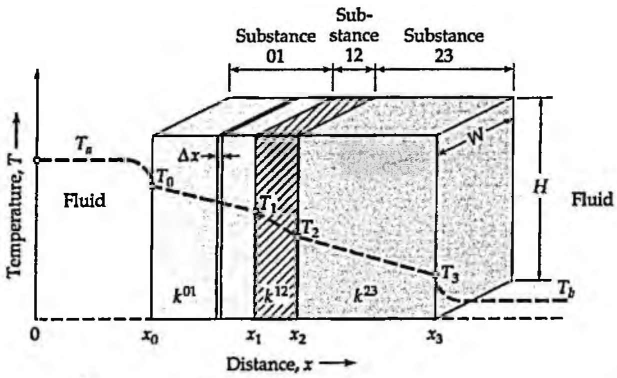
13. For the following materials at room temperature, (a) wood, parallel to axis, (b) wood, normal to axis, (c) graphite, and (d) copper, place them in the order of increasing thermal conductivity.
(A) (c)(b)(a)(d), (B) (a)(b)(c)(d), (C) (d)(c)(b)(a), (D) (b)(a)(c)(d).

14. For the following materials, (a) air (298 K, 1 atm), (b) aluminum (298 K), (c) H_2O (298 K), and (d) Hg (298 K), place them in the order of increasing Prandtl number.
(A) (d)(a)(c)(b), (B) (a)(d)(c)(b), (C) (d)(c)(b)(a), (D) (b)(a)(c)(d).

15. Consider an electric wire of radius R. The electric current generates heat at a rate per unit volume of Se. The surface of the wire is coated with an insulating material of thickness d and thermal conductivity k. The surface of the wire is maintained at temperature T_0. At steady state, what is the relationship between the heat flow rate normal to the interface of the wire and insulating material (A) and the heat generation rate by the electric current (B)?
(A) A=B, (B) A>B, (C) A<B, (D) cannot be determined.

16. Consider a Newtonian fluid of constant density and viscosity, flowing between two large plates separated by a distance b. The fluid flow is driven by the motion of the upper plate at a constant velocity V. The temperature of the lower plate (located at x=0) is maintained at T_0 and that of the upper plate at T_b (located at x=b). Let Br be the Brinkman number of the fluid flow. Under what conditions will the temperature distribution be nonlinear?
(A) Br=0, (B) Br<0, (C) Br>0, (D) any Br.

17. Consider the following steady state temperature profile in a laminated system (Fig. 10.6-1 of "Transport Phenomena" by R.B. Bird, W.E. Stewart, E.N. Lightfoot, 2nd ed., Wiley, 2002). Which of the following statements is true?



(A) $k^{01} > k^{12} > k^{23}$, (B) $k^{12}\frac{dT}{dx} > k^{23}\frac{dT}{dx}$ at $x = x_2$, (C) $k^{12}\frac{dT}{dx} = k^{23}\frac{dT}{dx}$ at $x = x_2$, (D) $k^{12} > k^{23} > k^{01}$.

18. Which of the following statements is wrong?
(A) Eddy thermal conductivity is not a physical property of a fluid.
(B) Eddy thermal conductivity depends on turbulent intensity of the flow.
(C) Prandtl number of heat transfer is analogous to Schmidt number of mass transfer.
(D) Chilton-Colburn analogy applies to laminar tube flow.

19. A solid material occupying the space from y=0 to $y=\infty$ is initially at temperature T_0. At time t=0, the surface at y=0 is suddenly raised to temperature T_1 and maintained at that temperature for t>0. (α and k are the thermal diffusivity and thermal conductivity of the material, respectively.) Which of the following statements is ***correct***?
(A) The temperature profile reaches a steady state after a long time.
(B) The normal heat flux at y=0 decreases with increasing time.
(C) The thermal boundary layer thickness decreases with increasing time.
(D) The temperature will be raised faster if the material possesses a smaller thermal diffusivity.

20. Consider a molten steel of velocity v_∞ and temperature T_∞ flowing over a flat plate fixed in space with its temperature maintained at T_S. There will be generated momentum and thermal boundary layers along the plate. What is the relationship between the thicknesses of the momentum boundary layer δ and thermal boundary layer δ_T ?

(A) $\delta = \delta_T$; (B) $\delta > \delta_T$; (C) $\delta < \delta_T$; (D) none of the above.

Problem 3 (10%)

Multiple choice: Pick only one answer for each question. Each problem is 2 points.

A hot stream with flow rate of 1 kg min^{-1} and heat capacity of 2 kJ $kg^{-1}K^{-1}$ must be cooled from 400 K to 300 K. A cold stream with flow rate of 2 kg min^{-1} and heat capacity of 1 $kg^{-1}K^{-1}$ is available at 300 K. If a temperature difference of 10 K must exist to drive effective heat transfer,

21. What is outlet temperature of the cold stream if e=heat exchanged is carried out in a countercurrent manner?
(A) 310 K, (B) 345 K, (C) 350 K, (D) 355 K, (E) 390 K

22. What is outlet temperature of the hot stream if heat exchanged is carried out in a countercurrent manner?
(A) 310 K, (B) 345 K, (C) 350 K, (D) 355 K, (E) 390 K

23. What is outlet temperature of the cold stream if heat exchanged is carried out in a cocurrent manner?
(A) 310 K, (B) 345 K, (C) 350 K, (D) 355 K, (E) 390 K

24. What is outlet temperature of the hot stream if the heat exchanged is carried out in a cocurrent manner?
(A) 310 K, (B) 345 K, (C) 350 K, (D) 355 K, (E) 390 K

25. What is maximum heat exchanged between the two streams?
(A) 90 kJ/min, (B) 100 kJ/min, (C) 190 kJ/min, (D) 200 kJ/min, (E) None of the above

Problem 4 (10%)

Multiple choice: Pick only one answer for each question. Each problem is 2 points.

25000 kg/hr of saturated steam at 224°C is to be condensed. The latent heat is 1839 kJ/kg. Cooling water is available at 20°C. The outlet water temperature is limited to 45 °C. The specific heat capacity is 4.18 kJ/kg.

26. Assuming that there is no heat loss to the surrounding, how much water is required?
(A) >25000 kg/hr
(B) <25000 kg/hr
(C) =25000 kg/hr
(D) None of above
(E) All of above

27. What is the log mean temperature difference (LMTD) of the heat exchange?
(A) =204 K
(B) =189 K
(C) =196.5 K
(D) >196.5 K and <204 K

(E) <196.5 K and >189 K

28. If 40000 kg/hr of cooling water was used, which of the following is possible?
(A) Heat is lost to the surrounding
(B) The outlet temperature of cooling is higher than 45°C
(C) The outlet temperature of cooling is less than 20°C
(D) None of above
(E) All of above

29. If 45000 kg/hr of cooling water was used, which of the following is possible?
(A) Heat is lost to the surrounding
(B) The outlet temperature of cooling is higher than 45°C
(C) The outlet temperature of cooling is less than 20°C
(D) None of above
(E) All of above

30. Assuming that there is no heat loss to the surrounding, and the overall heat transfer coefficient in the system is 1000 W/(m^2·K) how much heat exchange area is required?
(A) ~234 m^2
(B) ~23408 m^2
(C) ~65 m^2
(D) ~842 m^2
(E) None of above

Problem 5 (5%)

A coal particle burns in air at 1145 K, 1 atm. The process is limited by diffusion of the oxygen counterflow to the CO_2 and CO that are formed at the particle surface where

$$3C + 2.5O_2 \rightarrow CO + 2CO_2$$

Assume that the coal is pure carbon with a density of 1280 kg/m^3 and that the particle is spherical with an initial diameter of 0.015 cm. Air (21 mol% oxygen, 79 mol% nitrogen) exists several diameters away from the sphere. Provide the answers for the underlined blanks in the following questions.

(a) N_{COr} = ___N_{O2r} (where N_{Ar} is the molar flux of species A along the radial direction) *(2%)*

(b) The mole fraction of oxygen at the particle surface in the gas phase is ___. *(3%)*

Problem 6 (15%)

A stationary liquid layer of B is bounded by planes $z = 0$ (a solid wall) and $z = b$ (a gas-liquid interface). At these planes the concentration of A is C_{A0} and C_{Ab}, respectively. The diffusivity D_{AB} is a function of the concentration of A, i.e., $D_{AB} = D_{AB}(C_A)$.

(a) Write down the differential equation that can be used to derive the steady-state concentration distribution. *(2%)*

(b) Show that the concentration distribution is given by *(4%)*

$$\frac{\int_{C_A}^{C_{A0}} D_{AB}(C_A)\ dC_A}{\int_{C_{Ab}}^{C_{A0}} D_{AB}(C_A)\ dC_A} = \frac{z}{b}$$

(c) Show that the molar flux at the solid-liquid interface is *(4%)*

$$N_{Az}|_{z=0} = \frac{1}{b}\int_{C_{Ab}}^{C_{A0}} D_{AB}(C_A)\ dC_A$$

(d) Now assume that the diffusivity can be expressed as a Taylor series in the concentration

$$D_{AB}(C_A) = \bar{D}_{AB}[1 + \beta_1(C_A - \bar{C}_A) + \beta_2(C_A - \bar{C}_A)^2 + \cdots]$$

In which $\bar{C}_A = \frac{1}{2}(C_{A0} + C_{Ab})$ and $\bar{D}_{AB} = D_{AB}(\bar{C}_A)$. Then show that *(5%)*

$$N_{Az}|_{z=0} = \frac{\bar{D}_{AB}}{b}(C_{A0} - C_{Ab})[1 + \frac{1}{12}\beta_2(C_{A0} - C_{Ab})^2 + \cdots]$$

Problem 7 (15%)

1. CO_2 capture via chemical adsorption is studied (i.e., at a temperature of 450°C) in a small fixed-bed system (2 inches in diameter) with 200 grams of CaO particles, corresponding to a bed length of 10 cm.

Feed conditions are described as followed: the concentration of CO_2 in the feed (C_0; s gas feed) is 0.001 mol/L, and the flow rate of the feed is 2.0 L/min. We assume the density of the flow is constant in the whole process.

A CO_2 non-destructive infrared spectrometer (ND-IR) is employed to measure the concentration of CO_2 downstream (i.e., at the exit of the fixed-bed system) versus elution time (t). According to the results of the experiment, the concentration of CO_2, $C(t)$, equals to zero when $t = 0$ to 3 h (i.e., undetectable amount of CO_2). At $t = 4$ h, 5 h, and 6 h, surges of the CO_2 concentration to 0.0001 mol/L, 0.0005 mol/L, and 0.0009 mol/L, respectively, are identified. The concentration of CO_2 measured by the ND-IR is relatively constant at 0.001 mol/L when t is higher than 7 h.

Based on the information shown above, please:

(a) Draw a breakthrough curve in terms of relative concentration (i.e., $C(t)/C_0$) versus the length of the adsorption bed when $t = 2$ h versus $t = 9$ h. *(2%)*

(b) If the tolerance concentration of the CO_2 emitted is 5% of the original value, please calculate the saturation capacity of the adsorption bed (in terms of mol of CO_2 per gram of CaO particles). *(3%)*

(c) Followed on (b): If we want to have the breakthrough time increased by 10 times, what is the required length of the bed? (i.e., the tolerance concentration of the CO_2 emitted is 5% of the original value) *(3%)*

2. After a thorough evaluation of the case in Problem 7.1, the chemical adsorption system described above is concluded to be unfavorable for a long-term operation in the specified plant. Instead of using the chemical adsorption process shown above, we aim to operate the described system as a packed-bed gas absorption tower.

Our objective is to reduce the concentration of the emitted CO_2 to 10% of the original value. The original concentration of CO_2 in the gas flow, which is fed from the bottom of the packed-bed gas absorption tower, is 3 mol%. The total gas flow rate in the feed is 5 mol/s. Here the change of the total gas flow rate is negligible (i.e., sufficiently diluted).

We use CO_2-free de-ionized water (i.e., in the presence of the absorbent for CO_2 capture) as the solvent, which is fed from the top of the packed-bed gas absorption tower. The equilibrium correlation is expressed as $y_e = 30x_e$. Here, y_e and x_e are the equilibrium concentrations of CO_2 in the gas and liquid phases, respectively. The change of total liquid flow rate is negligible in this case.

According to the information shown above:

(a) Determine the minimum solvent rate (L_{min}) *(2%)*

(b) Determine the number of ideal stages required when $L = 3L_{min}$ *(3%)*

(c) Followed on (b): If the stage height is 1 m, and the estimated efficiency of the plate/stage is 10%: what is the required total height of this packed-bed gas absorption tower? Please show the detailed calculation along with answers. *(2%)*

Problem 8 (5%)

Spray drying is an attractive method for the fast removal of water of the aqueous colloid (i.e., to become dried, fine powder) under a relatively mild heat environment. Here, we attempt to use a spray dryer to convert the aqueous colloids of Particle A into dried, fine powders (i.e., in the form of nanoparticle clusters). Our goal is to remove 99.9995 wt% of the water from the sprayed droplet containing 20 wt% of solid particles. According to the mass balance, the composition of the droplet before drying is the same as the composition of the aqueous colloid.

In general, the drying process involved two steps: a constant rate zone and a falling rate zone. Due to the fast evaporation under the drying condition, 95% of water (i.e., related to the critical free moisture content) is removed immediately. Hence the time required for the drying at the constant rate period is assumed to be negligible (i.e., the rate of drying is 1 kg of $H_2O/(m^2 \cdot day)$ in this period). When the water content is below 5% of the original amount (i.e., the falling-rate period), the drying rate is assumed to be linearly proportional to the free moisture content. Based on the surface area measurement, the surface area of the dry solid Particle A available for drying is 1.0 m^2/g of solid Particle A.

Based on the information shown above:

(a) Draw a plot of drying rate (unit: kg of $H_2O/(m^2 \cdot day)$) versus free moisture content (unit: kg of H_2O/kg of solid Particle A) for an individual droplet, with detailed calculation. *(2%)*

(b) Determine the total time of drying required (unit: second) with detailed calculation. *(3%)*