1. Considering an endangered species, snowshoe hare, in the land of Hyrule. The natural growth rate of the hare is proportional to the population of the hare with the growth rate coefficient α. The natural death rate of the hare is also proportional to the population of the hare with the growth rate coefficient β. Unfortunately, $\beta > \alpha > 0$.
   (a) Write down a different equation (DE) for the population of hares using the assumptions above. (5%)
   (b) To save the snowshoe hare from extinction, H_1 hares are released at $t = T_1$ (day). With the initial hare population of H_0, $y(0) = H_0$, please find the population of the hare versus time $y(t)$ before and after $t = T_1$. (10 %)
   (c) For another way to save the snowshoe hare from extinction, H_2 hares are released every day from $t = T_1$ to $t = T_2$. Same as (b), with the initial hare population of H_0, $y(0) = H_0$, please find the population of the hare versus time $y(t)$ after $t = T_1$. (10 %)
2. Solve the following ODE (10%)
   $(3y - 2xy^3)dx + (4x - 3x^2y^2)dy = 0$
3. Solve the following 2nd order ordinary differential equation (15%)
   $x^2y'' + 2xy' + 4y = 10\sin(\ln x)$, for $x > 0$.
   Please find $y(x)$ when $x = 1$, $y(x) = 3, y'(x) = 0$,
4. (a) Show that Bessel equation is a Sturm-Liouville equation. (10%)
   $x^2 y'' + xy' + (x^2 - n^2)y = 0$
   (b) Prove the orthogonality of Bessel functions. What is their weight function? (10%)
5. The current fraction of rental cars in Denver is 0.02 and the fraction outside Denver is 0.98. Every month 80% of the Denver cars stay in Denver and 20% leave. Also 5% of the outside cars come in.
   (a) What is the Markov transition matrix? Calculate the fraction of the Denver and outside cars after one month. (5%)
   (b) Calculate the fraction of the Denver and outside cars after 2, 3, 4, 5, and 10 months; then plot the results on a figure with the fraction of the Denver cars on x-axis and the outside cars on y-axis. (10%)
6. Transforming the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2}\right)$

   by introducing new independent variables ($v = x + ct$, $w = x - ct$) and the use of the chain rule. Then solve the new transformed partial differential equation with initial conditions $u(x, 0) = f(x)$ and $u_t(x, 0) = g(x)$. (15%)

試題隨卷繳回