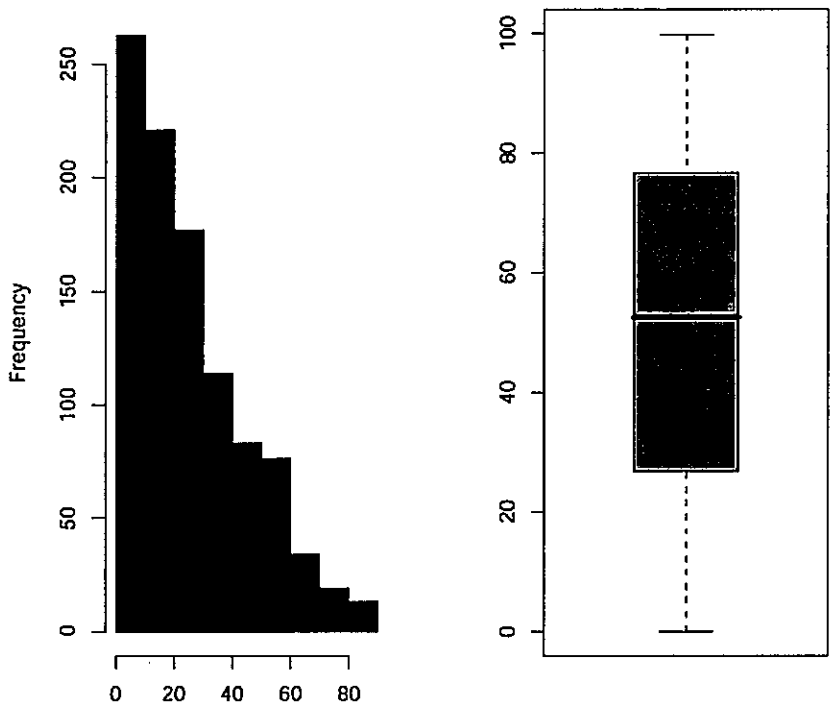
選擇題(單選題)(每題 5 分)　　※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

1. 以下是關於某作物種子千粒重(克)的一筆資料所繪的直方圖(Histogram)和盒形圖(Boxplot)。請問下列選項何者正確?



A. 樣本中位數約為 50 克。　　B. 樣本中位數介於 20 至 30 克之間。
C. 樣本均值高於樣本中位數。　　D. 樣本均值與樣本中位數非常接近。
E. 二張統計圖型的數據根本不可能是同筆資料。

2. 飲料工廠有兩條生產線，A 生產線產出 250c.c.的飲料且產品容量的標準差為 2c.c.，B 生產線產出 500c.c.的飲料且產品容量的標準差為 3c.c.，就產品容量的觀點而言：
A. A 生產線的品質較佳　　B. B 生產線的品質較佳.
C. 兩條生產線品質一樣.　　D. 難以斷論

3. 假定某一地區有 1% 的人感染新冠肺炎病毒，現有一快篩工具聲稱具有 95% 的敏感度(若有病，診斷結果呈陽性反應的機率)及 98% 的特異度(若沒病，診斷結果呈陰性反應的機率)。若某人篩檢結果為陽性，求他是確實感染病毒的機率約為多少？
A. 0.51　B. 0.20　C. 0.01　D. 0.02　E. 0.32

4. 研究發現某種子發芽率為 30%，若隨機抽取一樣本數為 n＝200 種子，並觀察其發芽個數，試問樣本比例的抽樣分佈為
A. $\mu = 20$，$\sigma = 0.3$ 的常態分佈　　B. $\mu = 3$，$\sigma = 1.96$ 的常態分佈
C. $\mu = 0.3$，$\sigma = 0.03$ 的常態分佈.　　D. 無法判斷

5. 承上題，樣本發芽比例會超過 0.35 的機率為何?
A. 0.1587　B. 0.9525　C. 0.0475　D. 1.0

6. 承上題，若此一隨機樣本觀測到 65 顆種子發芽，試問關於母體發芽比例的95%信賴區間估計為何?
A. [45.2%, 64.8%]　B. [26.6%, 38.4%]　C. [29.8%, 40.5%]　D. [54.2%, 55.8%]

7. 假設學生完成一份一小時生物統計學試卷所需時間(單位為小時比例)為一連續型隨機變數並服從以下密度函數(probability density function)

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$$f(x)=\begin{cases}Cx^2+x, 0\le x\le 1\\ 0, elsewhere\end{cases}$$

函數中 C 為常數，試問在 50 分鐘內完成試卷的機率為何?

A. 0.44　B. 0.54　C. 0.64　D. 0.73

8. 為了得知某一作物的平均株高，研究人員需進行收取樣本數為 n 的隨機樣本，假定此作物株高呈常態分布，其標準偏差為 10 公分。假若想做 95% 信賴區間估計並且區間長度為 2 公分，試問所需樣本數至少為何?

A. 97　B. 100　C. 385　D. 553

9. 某牌汽車製造商關心在 M 車型煞車系統發生的故障。此故障發生率不高，但在高速行駛時會引起嚴重的事故。假定每年有過此故障的汽車數目服從卜瓦松分布(Poission distribution)，平均每年會有 4 輛車發生此故障。試問在二年內超過 5 輛汽車發生此故障的機率約為多少？

A. 0.809　B. 0.785　C. 0.191　D. 0.215

10. 假設離散型隨機變數 X 和 Y 有以下聯合分布函數(joint probability mass function):

X \ Y	Y=0	Y=2	Y=4
X=0	0.1	0.1	0
X=2	0.1	0.4	0.1
X=4	0	0.1	0.1

試問隨機變數 X 和 Y 的相關係數(correlation coefficient)約為多少？

A. -0.8　B. -0.5　C. 0　D. 0.5　E. 0.8

11. According to the latest report which is about the deaths after the COVID-19 vaccine. There are about 60 deaths per million (10^6) with the AZ vaccine and about 40 with the Moderna vaccine. We want to know whether there is sufficient evidence that the two vaccines have different mortality rates.

(a) (5 points) Assume p_1 is the mortality rate of AZ vaccine and p_2 is the mortality rate of Moderna vaccine. Write down the null hypothesis H_0:_____ and alternative hypothesis H_1:_____

(b) (5 points) When significant level is 0.05, show the test statistic and your testing result? (using $1-\frac{40}{10^6}\approx 1, 1-\frac{60}{10^6}\approx 1, and\ 1-\frac{100}{2\times 10^6}\approx 1$)

12. Assume the age of student in a school follow Normal distribution $N(\mu,\sigma^2=4)$. When a random sample with size 16 have mean $\bar{X}=21$, test $H_0: \mu=\mu_0=20\ vs. H_1: \mu=\mu_1(>20)$

(a) (5 points) Using testing statistic $Z=\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$. Then, when $\bar{X}$>c, reject H_0 with significant level 0.05. find c ?

(b) (5 points) Show your testing result and explain that

(c) (5 points) When $H_1: \mu=\mu_1$ is true, find the type II error (expressed as the function of n, μ_0, μ_1, σ)

Continuous, next questions are "multiple-choice item", please choose the correct one from the following options. A: larger　B: smaller　C: no different　D: uncertain

(d) (2 points) When only μ_1 become larger, then type II error will be?

(e) (2 points) When only sample size (n) become larger, then type II error will be?

(f) (2 points) When only variance (σ^2) become larger, then type II error will be?

(g) (2 points) When only significant level become larger, then type II error will be?

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(h) (2 points) When only $\bar{X}$ become smaller, then type II error will be?

(i) (2 points) When type I error become larger, then type II error will be?

13. The result of a simple linear regression analysis is shown below

```
Call: lm(formula = y ~ x)
Residuals:
   Min      1Q  Median      3Q     Max
-8.6625 -3.9200  0.0525  3.0275  8.0076

Coefficients:
             Estimate   Std. Error  t value   Pr(>|t|)
(Intercept)  276.5626     2.4186    114.35    < 2× 10^-16 ***
x             -3.8900     0.3457    -11.25     4× 10^-9  ***

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.861 on 18 degrees of freedom
Multiple R-squared:  0.8755,    Adjusted R-squared:  0.8686
F-statistic with 1 and 18 DF
```

According to the above information, fulfill the ANOVA table (1 points for 1 cell)

	DF	SS	MS	F	p-value
Regression	(a)	2992.39	(e)	(g)	(h)
Error	(b)	425.36	(f)		
Total	(c)	(d)			

(i) (1 points) Show the estimated standard error of Y.

(j) (4 points)When residual $e_i = Y_i - \hat{Y}_i$, where $\widehat{Y_i} = \widehat{\beta_0} + \widehat{\beta_1} X_i$, then $\sum_{i=1}^{n} e_i = ?$, $\sum_{i=1}^{n} x_i e_i = ?$

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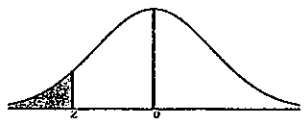
表一、標準常態分佈的累積機率表



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

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表二、卜瓦松分布 (Poission distribution) 的累積機率表

x	α 0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	1.0000	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	1.0000	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8	1.0000	1.0000	1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9972	0.9933	0.9863
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9976	0.9945
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

x	α 5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
0	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000
1	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005
23	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028
3	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103
4	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293
5	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671
6	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301
7	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202
8	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328
9	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579
10	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830
11	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968
12	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916
13	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645
14	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165
15	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513
16	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730
17	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857
18	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928
19	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965
20	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

試題隨卷繳回