國立交通大學 101 學年度碩士班考試入學試題

科目：線性代數與機率(1211) 考試日期:101 年 2 月 17 日 第 2 節

系所班別：電機工程學系 組別：電機系 第 1 頁,共 2 頁

【可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. (15%) Please determine whether the following statements are TRUE or FALSE. No explanations are needed.

   (a) (3%) Suppose $A\mathbf{x} = \mathbf{b}$ is a consistent system and $\mathbf{v}_1$ and $\mathbf{v}_2$ are two solutions, where A is an $m \times n$ matrix. Then $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$ must be a solution for the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

   (b) (3%) Let A and B be $m \times n$ matrices. If $A\mathbf{x} = B\mathbf{x}$ for all $\mathbf{x} \in \mathcal{R}^n$, then A and B are equal.

   (c) (3%) Let A be an $n \times n$ matrix. If A is invertible, then any subset of $\mathcal{R}^n$ formed by the columns vectors of A is linearly independent.

   (d) (3%) Let $\mathcal{S} = \{\mathbf{a}_1, \mathbf{a}_2, \cdots \mathbf{a}_k\}$ be a subset of $\mathcal{R}^n$ and $T : \mathcal{R}^n \to \mathcal{R}^m$ be a linear transformation. When $\{T(\mathbf{a}_1), T(\mathbf{a}_2), \cdots, T(\mathbf{a}_k)\}$ is linear independent, $\mathcal{S}$ is not necessarily linearly independent.

   (e) (3%) Let A be an $m \times n$ matrix. If the number of pivot numbers of A is less than m, then $A\mathbf{x} = \mathbf{b}$ has more than one solution.

2. (10%) Let $\mathcal{W}_1, \mathcal{W}_2, \cdots, \mathcal{W}_m$ be subspaces of $\mathcal{R}^n$ such that $\mathcal{W}_{i+1}$ is a subset of $\cap_{j=1}^{i} \mathcal{W}_j^{\perp}$, where $\mathcal{W}_j^{\perp}$ is the orthogonal complement of $\mathcal{W}_j$, for $i = 1, 2, \cdots, m-1$. Suppose the orthogonal projection matrix for $\mathcal{W}_i$ is P_i. Determine the orthogonal project matrix for $V = \{\mathbf{v} \in \mathcal{R}^n : \mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 + \cdots + \mathbf{v}_m,$ where $\mathbf{v}_i \in \mathcal{W}_i\}$.

3. (a) (8%) Find the eignevalues and eigenvectors for

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$$

   (b) (3%) How can you tell whether a matrix is invertible from its eigenvalues?

   (c) (3%) How can you tell whether two matrices are similar from their eigenvalues?

4. (a) (8%) For the following matrix, find the bases for its row space and nullspace.

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 4 & 5 & 6 & 3 \\ 3 & 4 & 4 & 1 \end{bmatrix}$$

   (b) (3%) In $\mathcal{R}^3$, is xy plane orthogonal to xz plane ? Explain it.

5. (12%) Let random variable X be uniformly distributed in [-2, 2]. Find $f_Y(y)$, the probability density function of $Y = X^2$ for $y > 0$.

6. (13%) Let X and Y be independent exponentially distributed random variables with common parameter λ. Find the probability density function of $Z = X + Y$.

7. (25%) Given a positive integer N, Let X and Y be discrete random variables with joint probability mass function (PMF)

$$p_{X,Y}(x, y) = \begin{cases} \dfrac{x^2 + y}{C}, & x \in Z^+, y \in Z^+, x + y \le N \\ 0, & \text{otherwise} \end{cases}$$

where Z^+ denotes the set of nonnegative integers and C is a normalizing constant.

(a) (10%) Find the marginal PMF of X in terms of N and C.

(b) (5%) If N=3, find C.

(c) (10%) If N=3 and $Z = X(N - X)$, find $p_Z(z)$, the PMF of Z, for all z.