國立政治大學 111 學年度 碩士暨碩士在職專班 招生考試試題

考 試 科 目	統計學	系 所 別	風險管理與保險學系 精算科學組	考 試 時 間	2 月 10 日 (四) 第 四 節

1. (30 pts) Please explain the following items.

   (a) (10 pts) The Rao-Cramer Inequality

   (b) (10 pts) The Rao-Blackwell Theorem

   (c) (10 pts) Chebyshev's Inequality and its proof

2. (15 pts) Let X_1, X_2, X_3, X_4 be mutually stochastically independent random variables, each with p.d.f.

$$f(x) = \begin{cases} e^{-x}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases} .$$

   Find the distribution of $Y = \min\{X_1, X_2, X_3, X_4\}$.

3. (15 pts) Let X be a random variable following $N(\mu, \sigma^2)$ and consider a transform $Y = e^X$.

   (a) (9 pts) Find the p.d.f. of Y.

   (b) (6 pts) Find the mean and the variance of Y.

4. (20 pts) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with one of two pdfs.

$$f(x;\theta) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, & -\infty < x < \infty\,, \quad \theta = 1\,; \\ \frac{1}{\pi(1+x^2)}, & -\infty < x < \infty\,, \quad \theta = 2\,. \end{cases}$$

   Find the maximum likelihood estimation (MLE) of θ.

5. (20 pts) Let X_1 and X_2 be the two independent exponential random variables with population means α_1 and α_2, respectively. Please show that the probability density function of the difference of two exponentials $Y = X_1 - X_2$, $f_Y(y)$, has a Laplace distribution with parameters α_1 and α_2.

$$f_Y(y) = \begin{cases} \frac{e^{\frac{y}{\alpha_2}}}{\alpha_1+\alpha_2}, & y \le 0 \\ \frac{e^{-\frac{y}{\alpha_1}}}{\alpha_1+\alpha_2}, & y > 0 \end{cases} .$$

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。