

考試科目	微積分	系所別	政治管理學院 精進班	考試時間	2月10日(四)第二節
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**Problem 1** (10 points)

Evaluate the integral

$$\int_0^1 \frac{(2+x)^2}{1+x^2} dx.$$

**Problem 2** (15 points) (5 % each)

- (a) Write down the general terms the MacLaurin series of  $\sin x$  and  $\sin^{-1}x$ .  
 (b) Find their radii of convergence.  
 (c) Find

$$\lim_{x \rightarrow 0} \frac{\sin x \cdot \sin^{-1}x - x^2}{x^6}.$$

**Problem 3** (15 points)Find the critical points of  $f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 2$ . Which of them give rise to maximum values? Minimum values? Saddle points?**Problem 4** (10 points)

Determine the convergence of the following series. (5 % each)

(a)

$$\sum_{n=0}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}.$$

(b)

$$\sum_{n=1}^{\infty} \sqrt{\sin \frac{1}{n^3}}.$$

考試科目	微積分	系所別	政大商學院(學院) 數學系	考試時間	2 月 10 日 (四) 第二節
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**Problem 5 (10 points)**

Evaluate the integral

$$\int \frac{(e^{3x} + 1)}{(e^{2x} + 1)^2} dx.$$

**Problem 6 (10 points)**

Compute

$$\int \frac{1}{e^{2x} + e^x + 1} dx.$$

**Problem 7 (10 points)**

A plane curve is defined by  $(x^2 + y^2)^2 = 2xy$ . (5 % each)

- (a) Find its polar coordinate equation, and draw the graph.  
 (b) Find the area of the plane region bounded by the curve.

**Problem 8 (20 points)**

Evaluate the limits. (10 % each)

(a)

$$\lim_{x \rightarrow \infty} (2^x + 3^x + 5^x)^{\frac{1}{x}}.$$

(b)

$$\lim_{x \rightarrow \infty} \frac{\sin\left(\frac{1}{\sqrt{x^2+1}}\right)}{\sqrt{x^2+2} - \sqrt{x^2-1}}.$$

備註

- 一、作答於試題上者，不予計分。  
 二、試題請隨卷繳交。