考試科目	統計方法	系所別	統計學系	考試時間	2月 9日(三) 第四節

1. (12%) A Center of Sleep Disorder conducted a survey to determine whether the hours of sleep per night are independent of age. A random sample of 500 individuals was asked two questions: (i) hours of sleep per night (with four options: fewer than 6, 6 to 6.9, 7 to 7.9, and 8 or more); (ii) age (younger than 30, and 30 or older). The data are given in the following table.

	Age	
Hours of sleep	Younger than 30	30 or older
fewer than 6	60	90
6 to 6.9	70	70
7 to 7.9	60	60
8 or more	50	40

(a) Using a 0.05 significance level, conduct a hypothesis test to determine whether the hours of sleep per night are independent of age. Please complete the hypothesis test. (10%)

(b) What is your interpretation about the result in (a)? (2%)

2. (20%) For testing $H_0: \mu \le 22$ v.s. $H_a: \mu > 22$, a random sample of 256 individuals is selected from the population. The population variance is 400. The power is 0.2005, denoted by $1 - \beta_{23.4}$, when the actual population mean is 23.4.

(a) What is the significance level adopted in the test? (12%)

(b) In such a hypothesis testing with a sample of 256 individuals, is it possible that the probability of type II error $\beta_{23.4}$ and the significance level decrease simultaneously? Please give your reason. (8%)

3. (35%) A realty company sells apartments in the Midwest of the United States. Since their customers are interested in the fee they have to pay for heating in winter, a research group in the company collected data and tried to build regression models to predict the heating cost (abbreviated to Heat.cost; denoted by Y). There were three independent variables: the mean temperature in winter (Temp, X_1), the centimeters of insulation in the attic (Insu, X_2), and the age (years) of the heater (Age, X_3). The research group considered the following regression analyses:

<u>Model 1</u>: $Y = \alpha + \beta_1 X_1 + \eta$,

<u>Model 2</u>: $Y = \alpha + \beta_3 X_3 + \delta$,

<u>Model 3</u>: $Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$,

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where η, δ, and ε are error terms. They collected data from 20 of their former customers for data analysis. First of all, the Model 1 is fitted based on the 10 elements given in the following table, and please answer Questions (a) and (b).

Customer	Y	X_1
1	250	−5
2	360	−9
3	165	−5
4	43	9
5	92	11
6	200	−8
7	355	−19
8	290	−21
9	230	−13
10	120	6

(a) Find the least squares estimates of α and β_1 under Model 1. (12%)
(b) Compute the coefficient of determination, R^2, for Model 1. (12%)

An output from a computer software for fitting Model 3 to the data is displayed in the followings. Using the output to answer Questions (c) and (d).

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  228.833     40.547   5.644 3.66e-05 ***
Temp          -8.348      1.361  -6.134 1.44e-05 ***
Insu          -6.098      1.872  -3.258  0.00494 **
Age            5.751      3.937   1.461  0.16349
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 49.98 on 16 degrees of freedom
Multiple R-squared:  0.8123,    Adjusted R-squared:  0.7771
F-statistic: 23.08 on 3 and 16 DF,  p-value: 4.695e-06
```

The output for fitting Model 2 to the data is displayed in the followings.

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   86.184     48.656   1.771   0.0934 .
Age           17.009      6.303   2.699   0.0147 *
```

考 試 科 目	統計方法	系 所 別	統計學系	考 試 時 間	2 月 9 日(三) 第 四 節

(c) What is your interpretation about the result of the F-test for Model 3 ? (5%)

(d) Based on the fittings, the β_3 under Model 2 is significant while the β_3 under Model 3 is not, at a 0.05 significance level. Comment on the phenomenon (Is there any contradiction, or …?). (6%)

4. (10%) Consider the hypothesis test that $H_0: \mu \geq \mu_0$ v.s. $H_a: \mu < \mu_0$, where μ_0 is a fixed constant. The population variance is known as σ^2. Suppose that the significance level is set at α, and suppose further that the probability of type II error is β when the population mean is μ_{0a}, where μ_{0a} is a constant smaller than μ_0. Define z_α and z_β as numbers satisfying $P(Z \geq z_\alpha) = \alpha$ and $P(Z \geq z_\beta) = \beta$. Derive that the sample size should be $\frac{(z_\alpha + z_\beta)^2 \cdot \sigma^2}{(\mu_0 - \mu_{0a})^2}$ (and then round up).

5. (23%) A research study is conducted to compare the kilometers per liter of unleaded regular, mid-grade, and super premium gasolines. There are differences in the performances of different automobiles. Seven different automobiles (numbered in 1 to 7) are sampled randomly and tested. The experimental results, in kilometers per liter, are given in the following table.

	Gasolines		
Automobile	Regular	Mid-grade	Super Premium
1	26	26	29
2	28	25	28
3	26	28	30
4	29	28	30
5	31	30	35
6	28	29	33
7	28	30	32

(a) To answer the research question, an ANOVA is performed. What are the null and alternative hypotheses for the ANOVA? (2%)

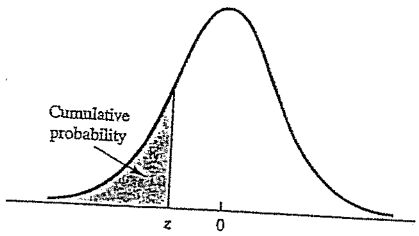
(b) Set up the ANOVA table. (17%)

(c) Using a 0.05 significance level, what is the conclusion? What is your interpretation about the result? (4%)

考試科目	統計方法	系所別	統計學系	考試時間	2月 9日(三) 第四節

Table of the standard Normal Distribution (its cumulative probabilities)

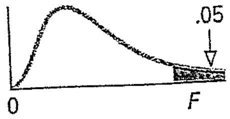
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



國立政治大學 111 學年度 碩士暨碩士在職專班 招生考試試題

考試科目	統計方法	系所別	統計學系	考試時間	2月 9日(三) 第四節

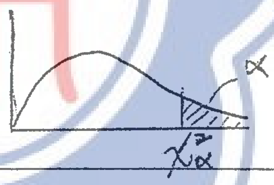
Table of the F Distribution ($\alpha = 0.05$)



Degrees of Freedom for the Numerator (columns); Degrees of Freedom for the Denominator (rows)

	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99

Table of the Chi-square Distribution



Degrees of Freedom	Area in Upper Tail .995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	.000	.000	.001	.004	.016	2.706	3.841	5.024	6.635	7.879
2	.010	.020	.051	.103	.211	4.605	5.991	7.378	9.210	10.597
3	.072	.115	.216	.352	.584	6.251	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	1.064	7.779	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	1.610	9.236	11.070	12.832	15.086	16.750
6	.676	.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.647	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。